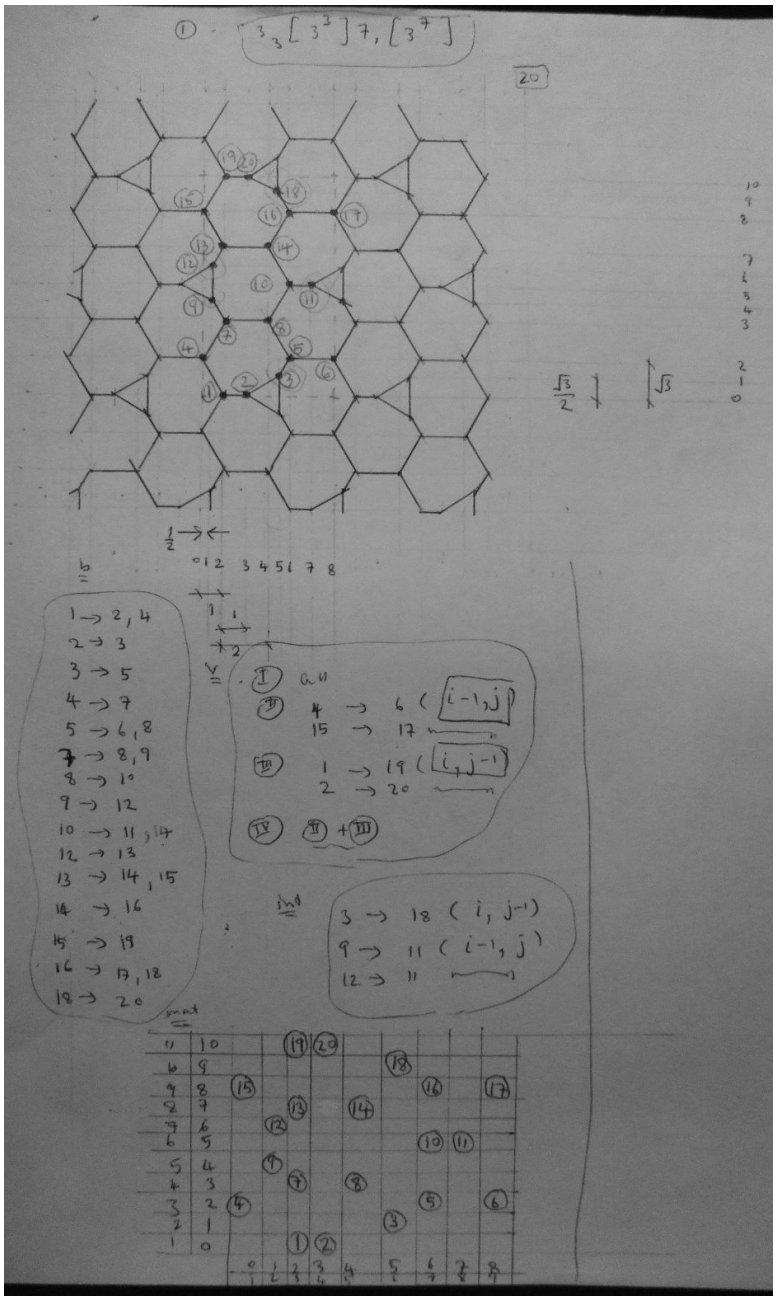
2-homohedral tiling sketches†

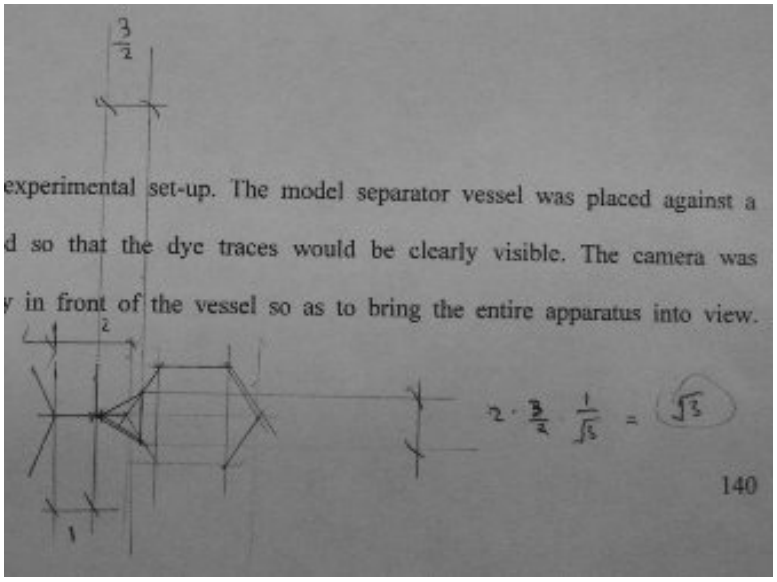
by

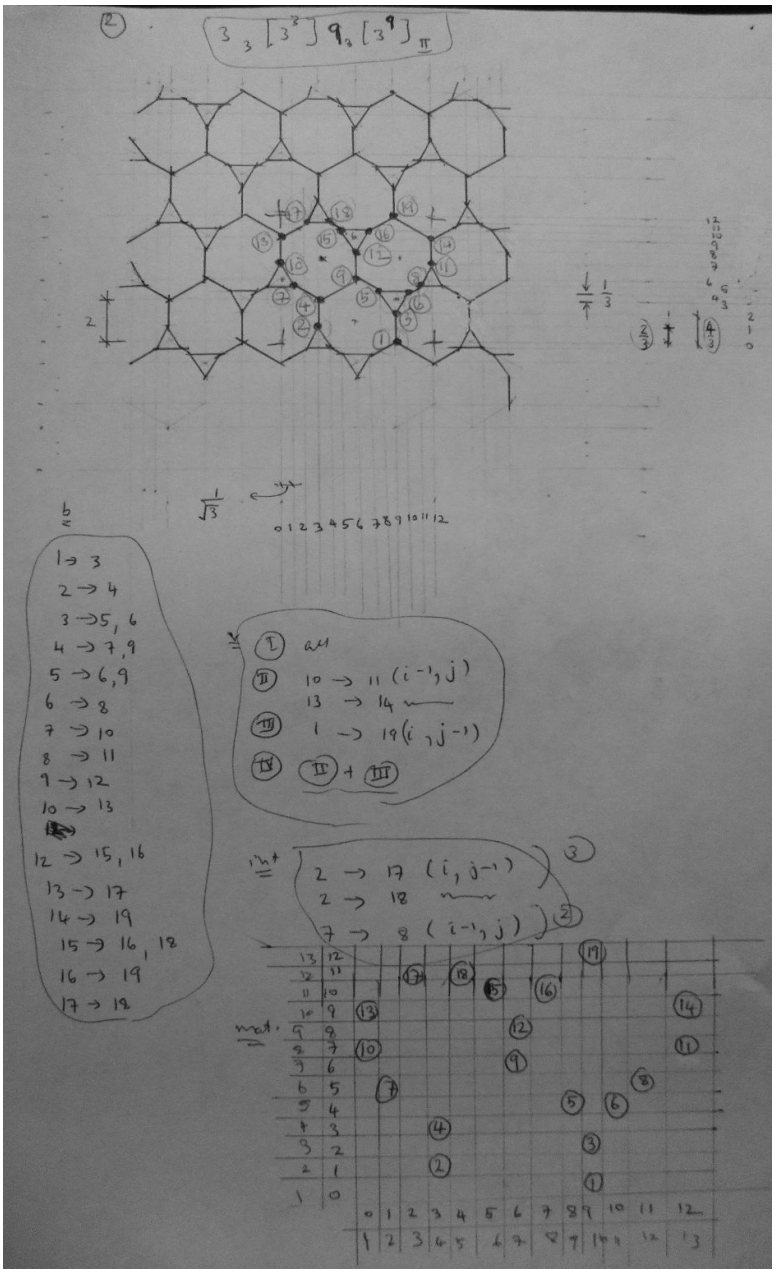
Kit Johnstone Tyabandha, Ph.D.

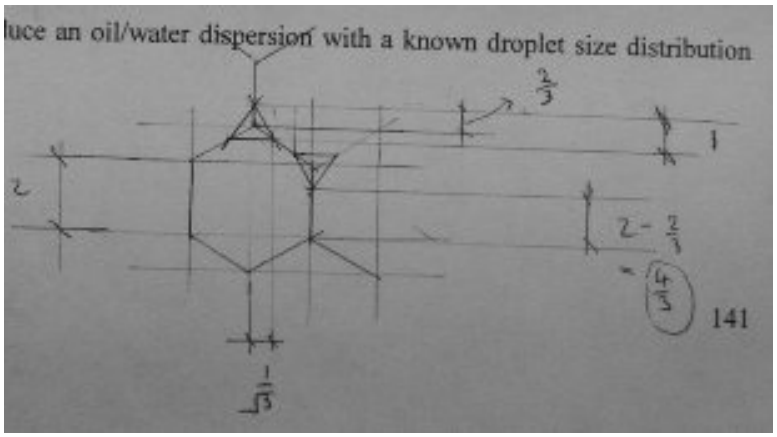
2002

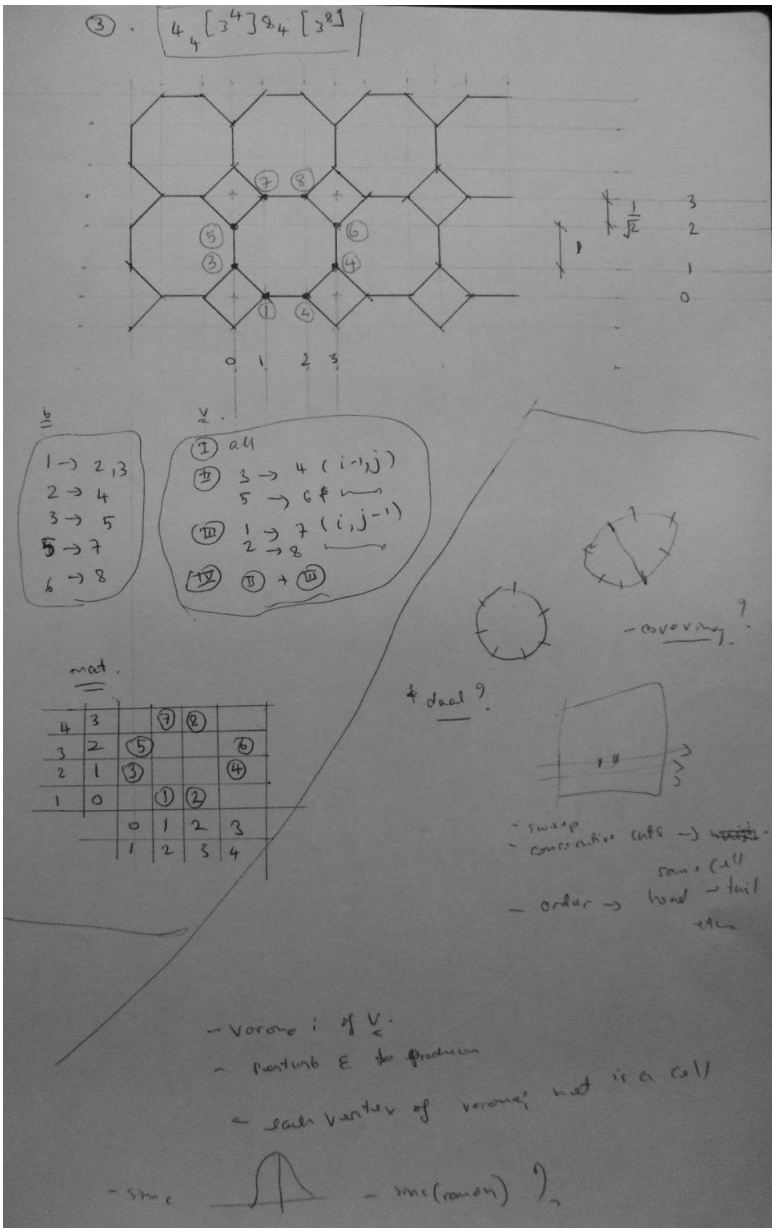
† Used in programmes included in my Ph.D. thesis, 2004, University of Manchester.

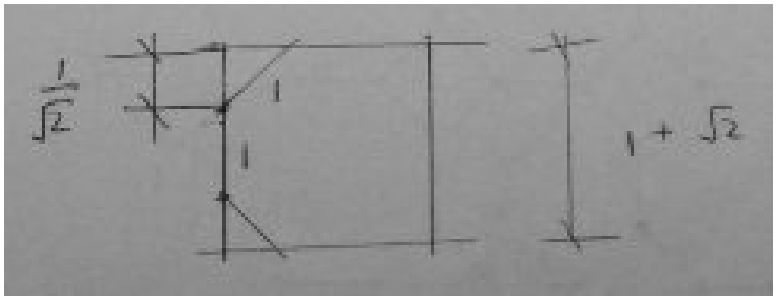

① · [3 3 [3 3] 7, [3 7]
20
1 → 2, 4
2 → 3
3 → 5
4 → 7
5 → 6, 8
7 → 8, 9
8 → 10
9 → 12
10 → 11, 14
12 → 13
13 → 14, 15
14 → 16
15 → 19
16 → 17, 18
18 → 20
4 → 6 (i-1, j)
15 → 17
1 → 19 (i, j-1)
2 → 20
3 → 18 (i, j-1)
9 → 11 (i-1, j)
12 → 11

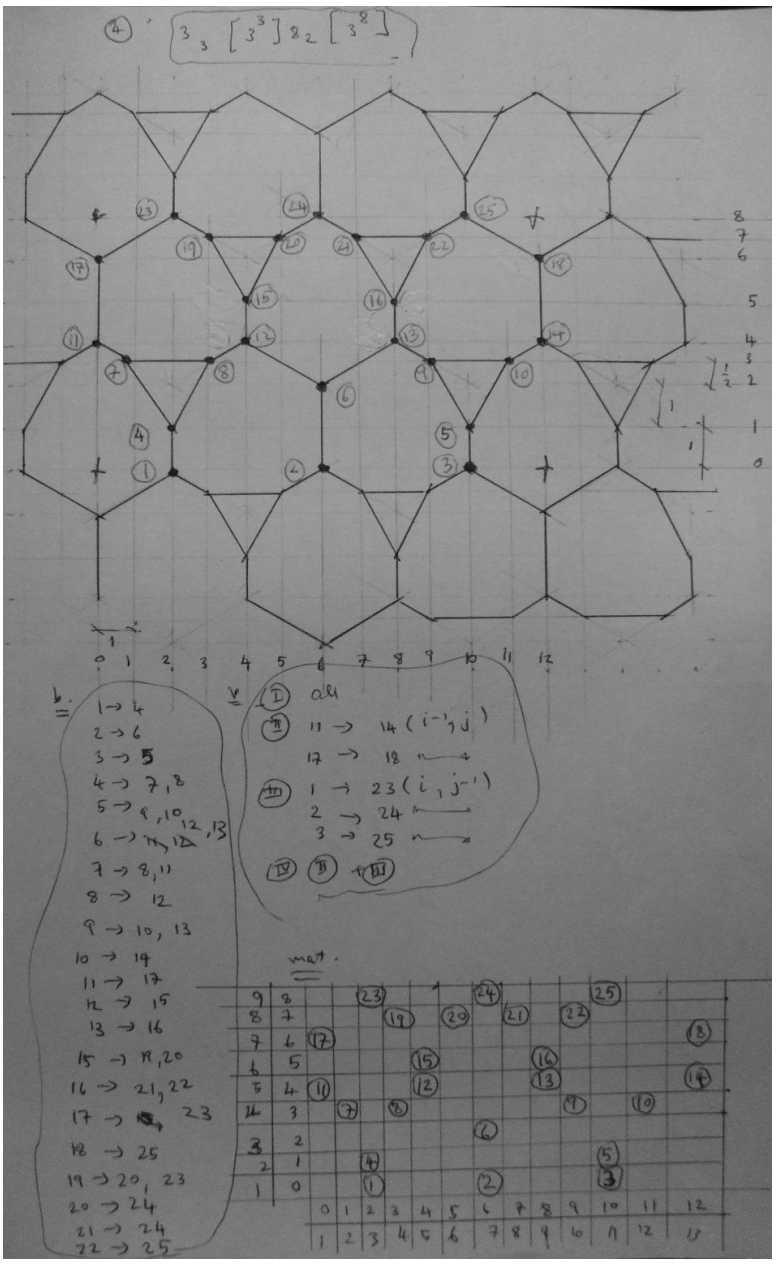

3/2
experimental set-up. The model separator vessel was placed against a
d so that the dye traces would be clearly visible. The camera was
y in front of the vessel so as to bring the entire apparatus into view.
2
1
2 · 3/2 1/√3 = √3
140

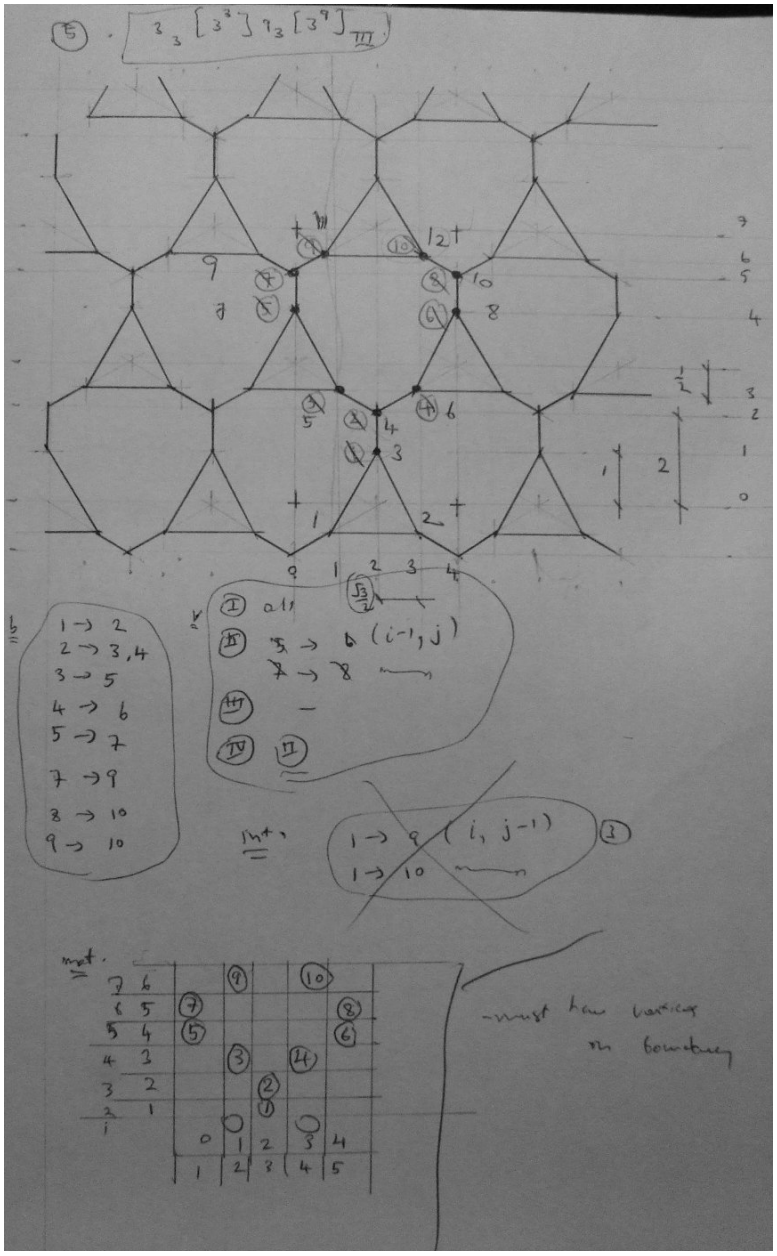

1 → 3
2 → 4
3 → 5, 6
4 → 7, 9
5 → 6, 9
6 → 8
7 → 10
8 → 11
9 → 12
10 → 13
12 → 15, 16
13 → 17
14 → 19
15 → 16, 18
16 → 19
17 → 18
I all
II 10 → 11 (i⁻¹, j)
13 → 14
III 1 → 19 (i, j⁻¹)
IV II + III
2 → 17 (i, j⁻¹)
2 → 18
7 → 8 (i⁻¹, j)

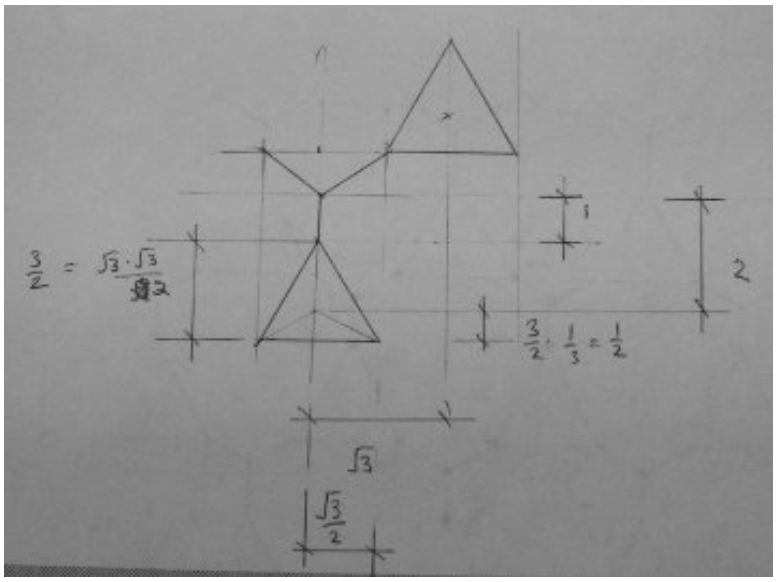

luce an oil/water dispersion with a known droplet size distribution
141

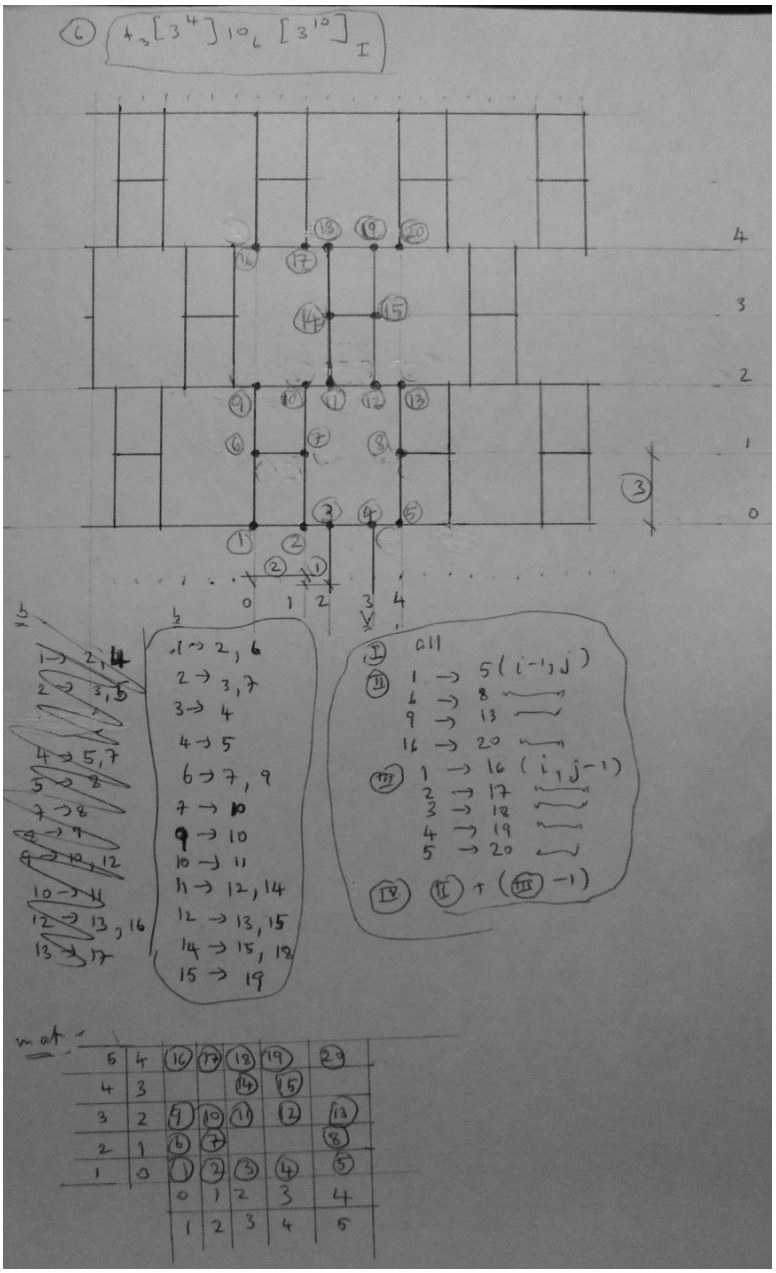

③ · 4 4 [3^4] 8 4 [3^2]
0 1 2 3
3
2
1
0
b
1 → 2,3
2 → 4
3 → 5
5 → 7
6 → 8
v
I all
II 3 → 4 (i-1,j)
5 → 6
III 1 → 7 (i,j-1)
2 → 8
IV II + III
mat.
4 3 7 8
3 2 5 6
2 1 3 4
1 0 1 2
0 1 2 3
1 2 3 4
& dual ?
- swap
- consecutive cuts → wrap
- order → row: call
head → tail
- Voronoi of V.
- perturb ε to produce
- each vertex of voronoi not is a cell
- sin(x)
- sin(random) ?

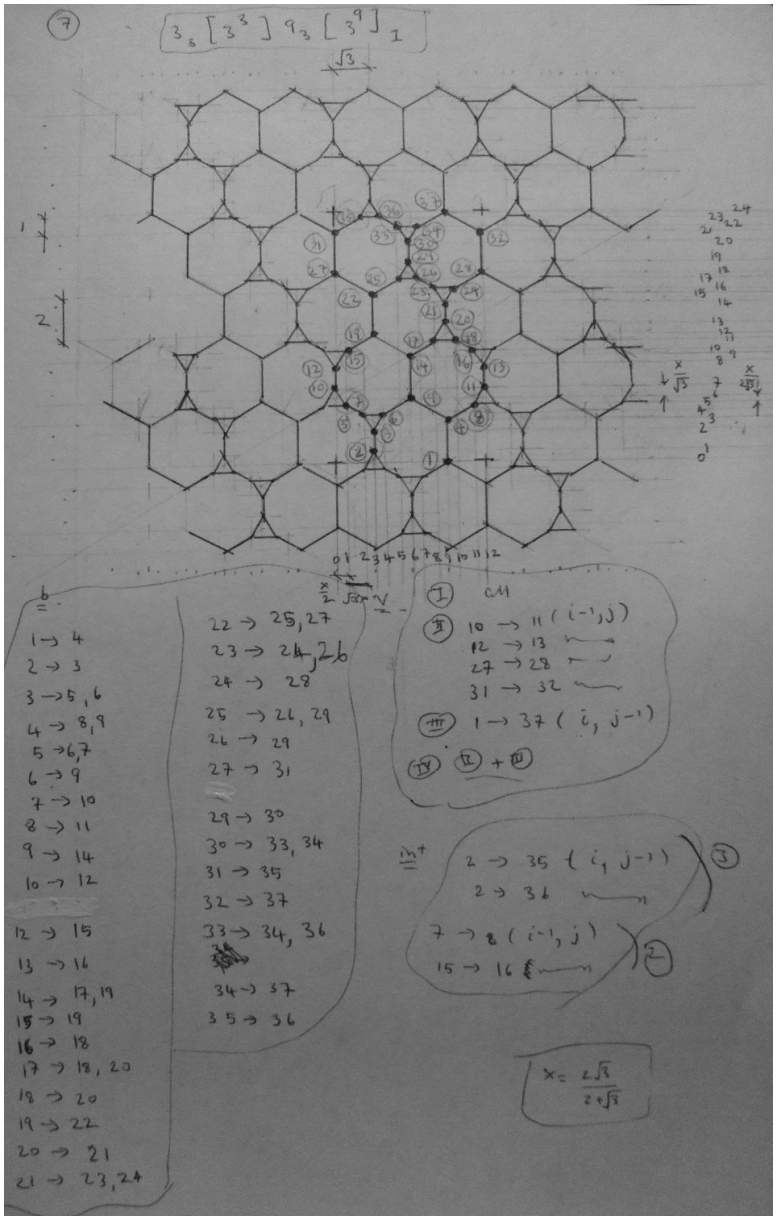

$\frac{1}{\sqrt{2}}$
1
1
$1+\sqrt{2}$

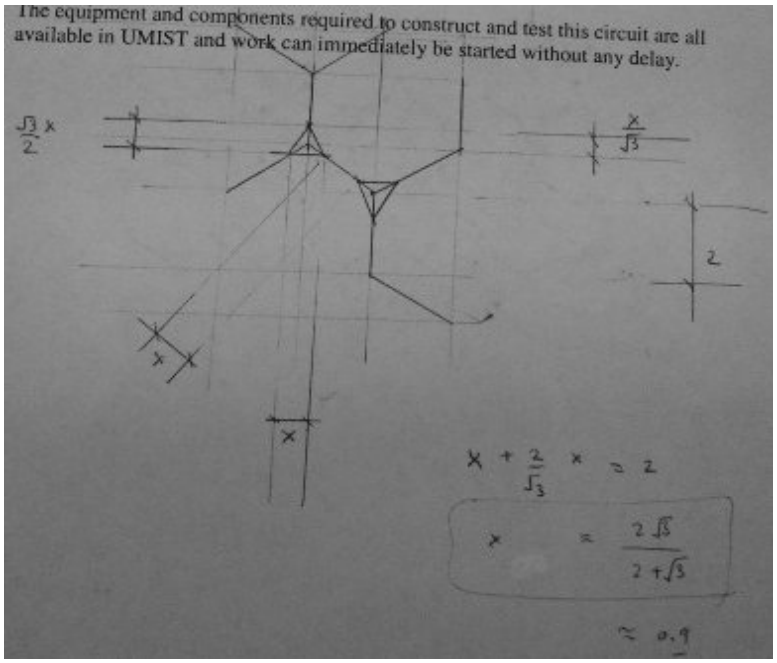

[3 3] 3 2 [3 3]
[3 8]
1 → 4
2 → 6
3 → 5
4 → 7, 8
5 → 8, 10, 12, 13
6 → 7, 12, 13
7 → 8, 11
8 → 12
9 → 10, 13
10 → 14
11 → 17
12 → 15
13 → 16
15 → 18, 20
16 → 21, 22
17 → 23
18 → 25
19 → 20, 23
20 → 24
21 → 24
22 → 25
all
11 → 14 (i⁻¹, j)
17 → 18
1 → 23 (i, j⁻¹)
2 → 24
3 → 25

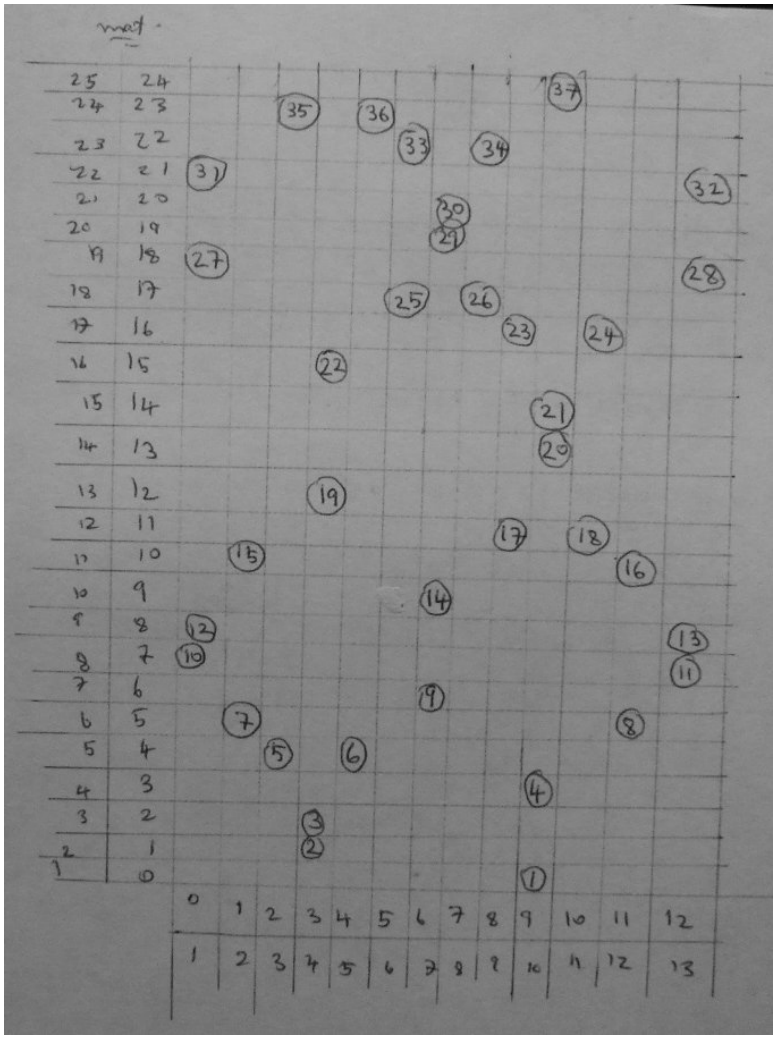

1 → 2
2 → 3,4
3 → 5
4 → 6
5 → 7
7 → 9
8 → 10
9 → 10
all
must have vertices on boundary

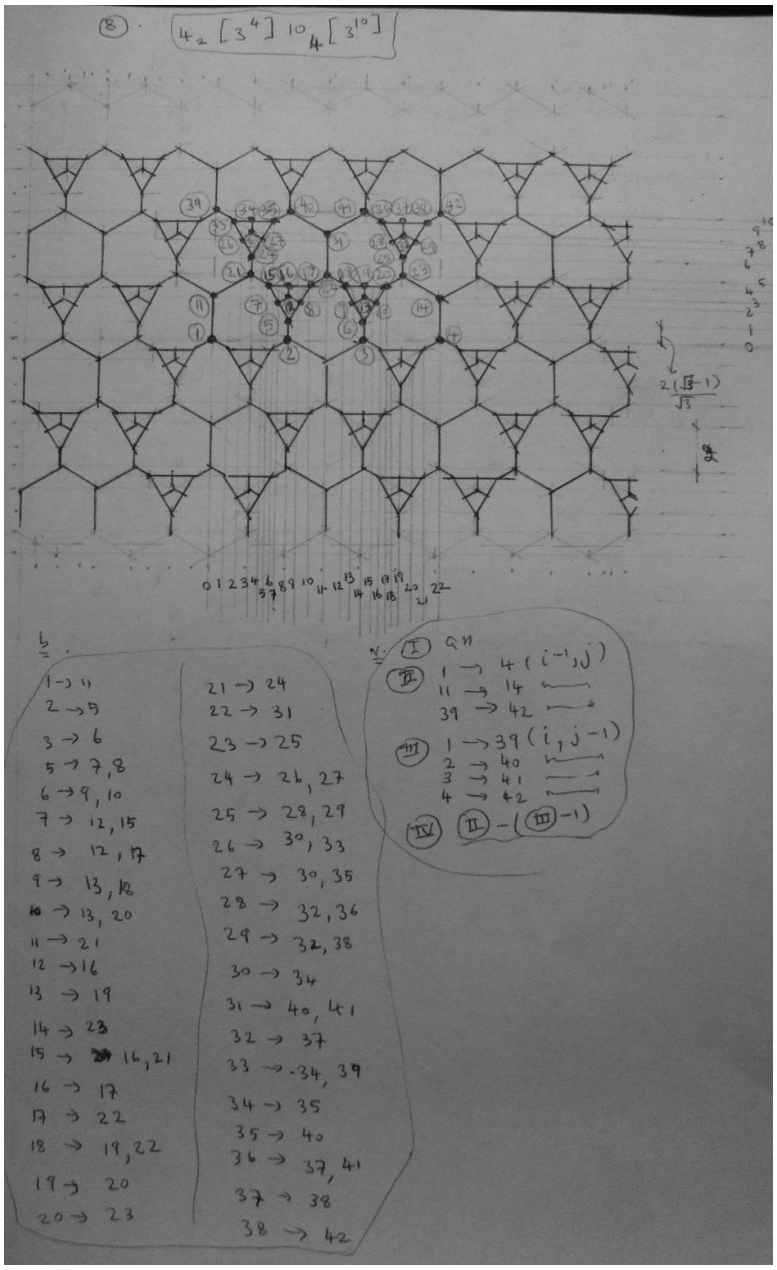

$\frac{3}{2} = \frac{\sqrt{3}\cdot\sqrt{3}}{2}$
1
2
$\frac{3}{2} \cdot \frac{1}{3} = \frac{1}{2}$
$\sqrt{3}$
$\frac{\sqrt{3}}{2}$

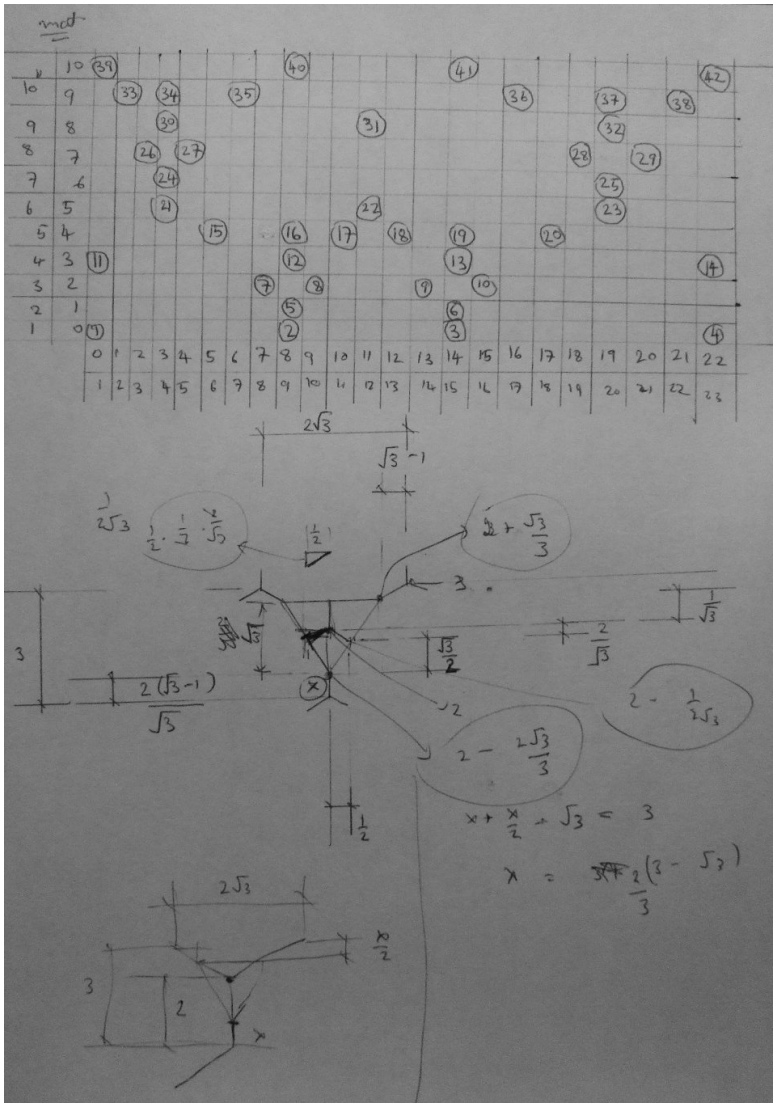

6
$4_3[3^4]10_6[3^{10}]_I$
all
I
II
III
IV
V
$i \to 5(i-1,j)$
$1 \to 16\ (i, j-1)$
$\to$ (III) -1)
mat

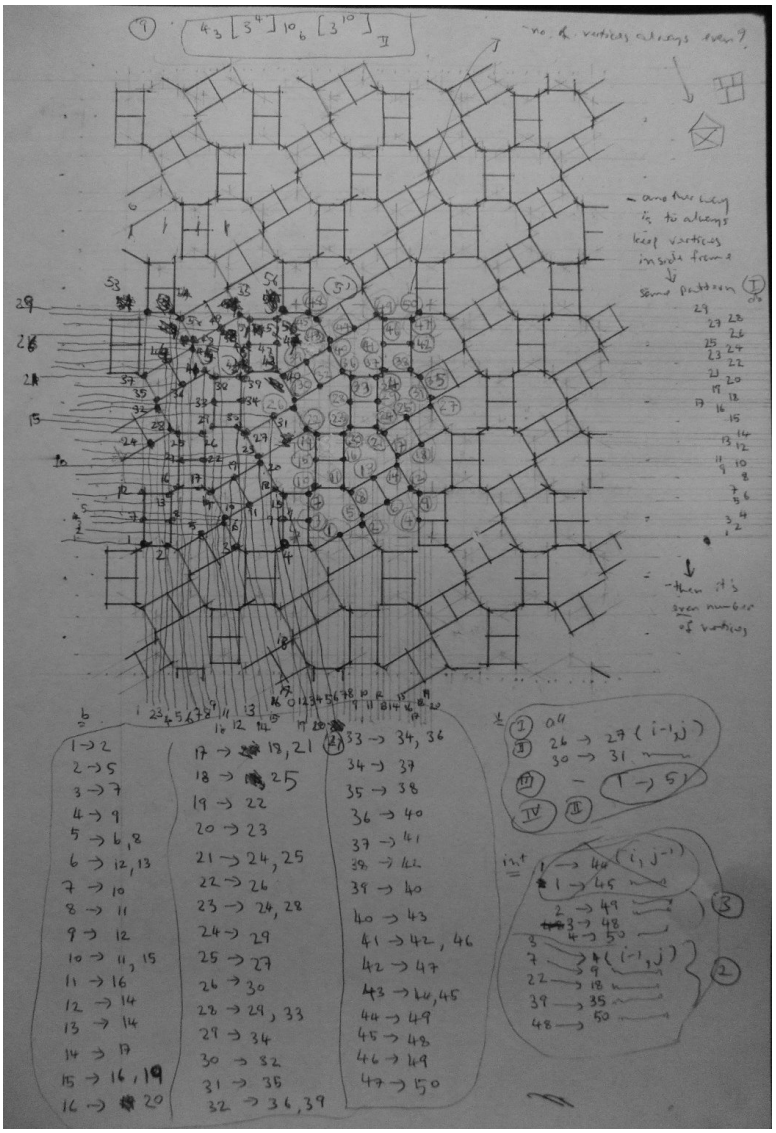

7
$3_6[3^3]\ 9_3[3^9]\ 1$
$\sqrt{3}$
1
2
$\frac{x}{\sqrt{3}}$
$\frac{x}{2\sqrt{3}}$
0 1 2 3 4 5 6 7 8 9 10 11 12
$\frac{x}{2}$ $\frac{x\sqrt{3}}{2}$
1 → 4
2 → 3
3 → 5, 6
4 → 8, 9
5 → 6, 7
6 → 9
7 → 10
8 → 11
9 → 14
10 → 12
12 → 15
13 → 16
14 → 17, 19
15 → 19
16 → 18
17 → 18, 20
18 → 20
19 → 22
20 → 21
21 → 23, 24
22 → 25, 27
23 → 24, 26
24 → 28
25 → 26, 29
26 → 29
27 → 31
29 → 30
30 → 33, 34
31 → 35
32 → 37
33 → 34, 36
34 → 37
35 → 36
I cm
II 10 → 11 (i-1, j)
12 → 13
27 → 28
31 → 32
III 1 → 37 (i, j-1)
IV II + III
2 → 35 (i, j-1)
2 → 34
7 → 8 (i-1, j)
15 → 16
$x = \frac{2\sqrt{3}}{2+\sqrt{3}}$

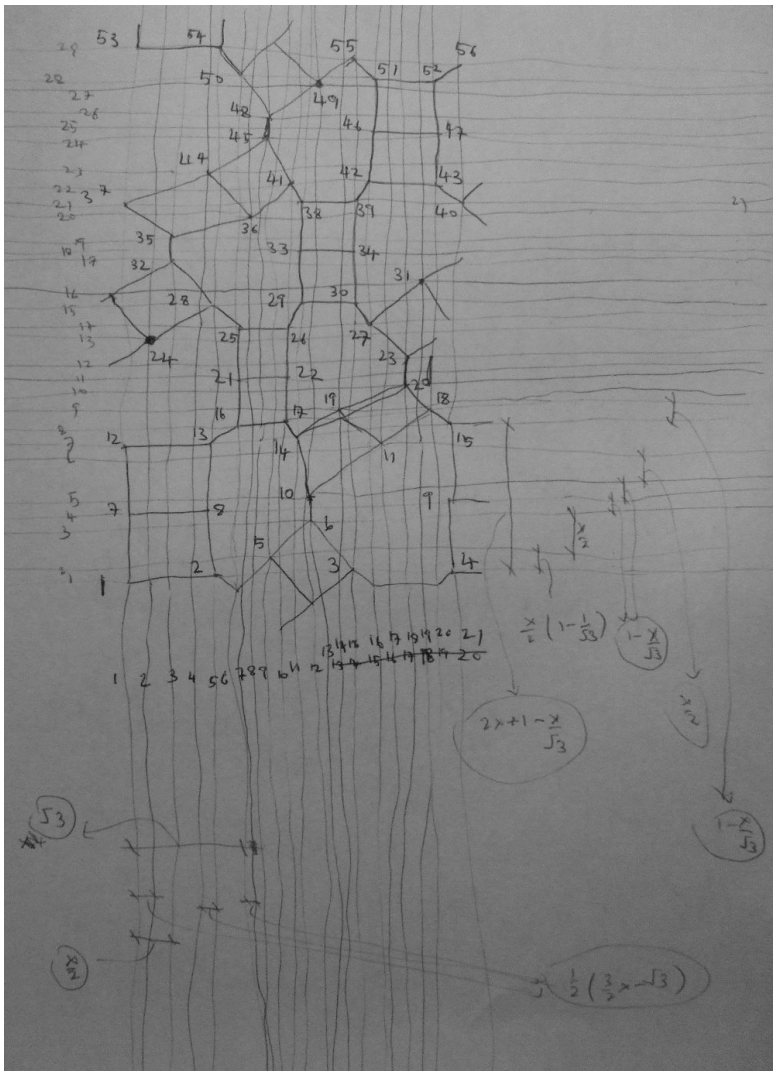

The equipment and components required to construct and test this circuit are all available in UMIST and work can immediately be started without any delay.
$\frac{\sqrt{3}}{2}x$
$\frac{x}{\sqrt{3}}$
2
x
x
$x + \frac{2}{\sqrt{3}} x = 2$
$x = \frac{2\sqrt{3}}{2+\sqrt{3}}$
≈ 0.9

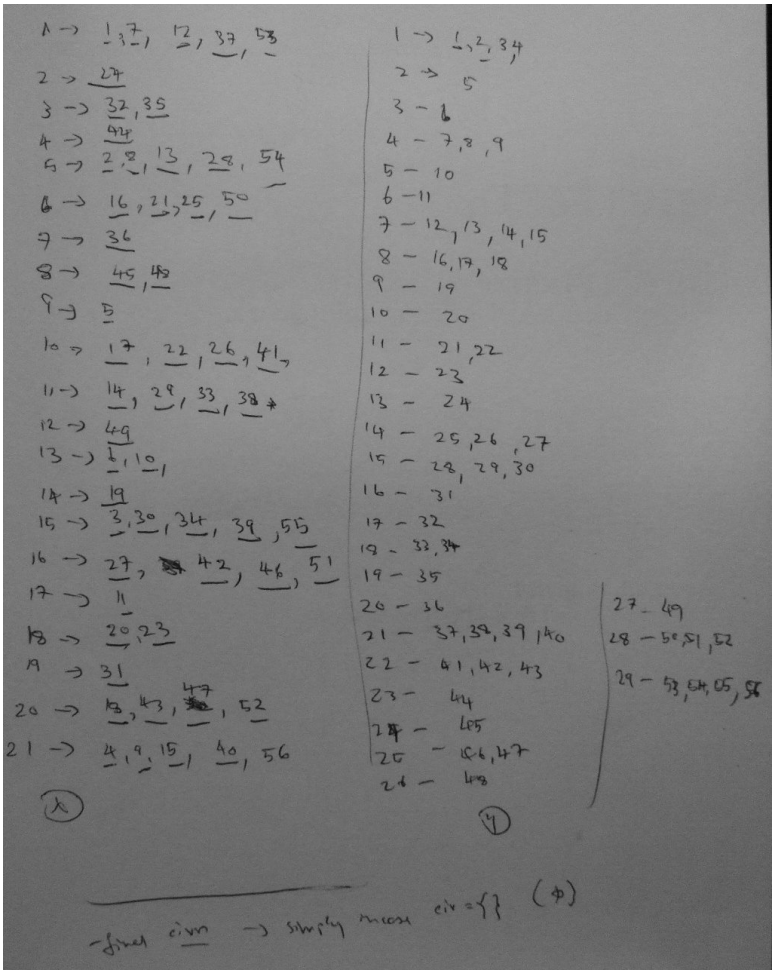


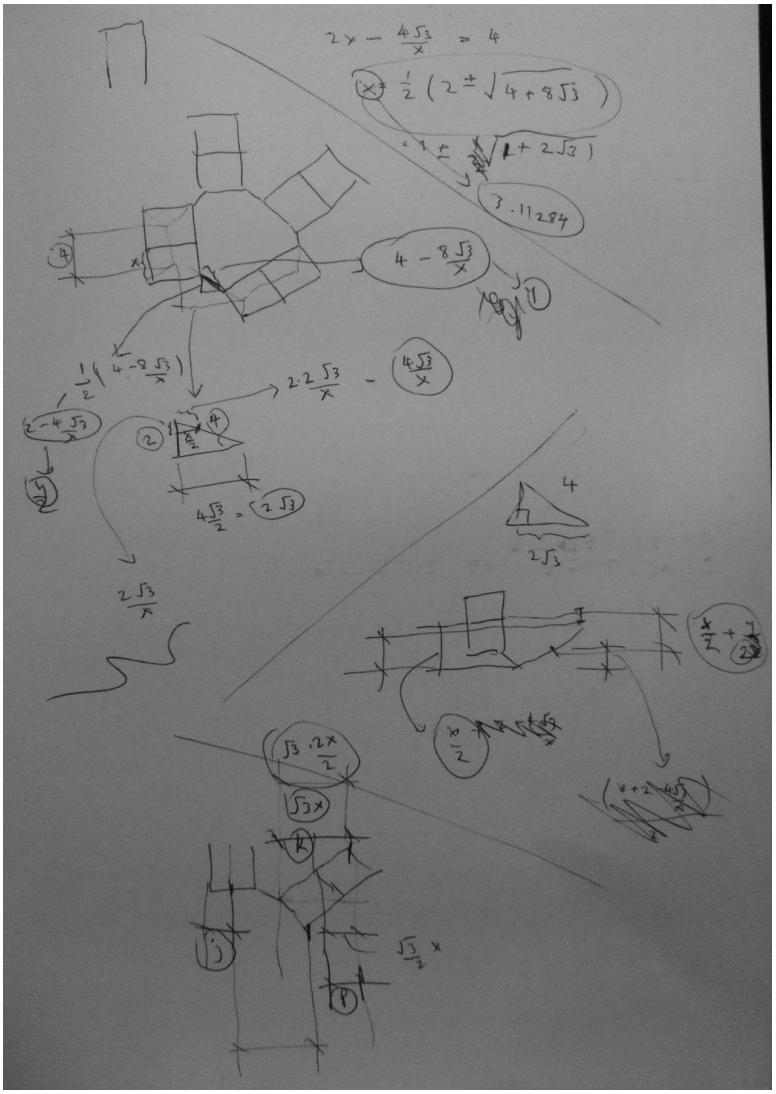

(8) $4_2[3^4]10_4[3^{10}]$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
1 → 11
2 → 5
3 → 6
5 → 7, 8
6 → 9, 10
7 → 12, 15
8 → 12, 17
9 → 13, 18
10 → 13, 20
11 → 21
12 → 16
13 → 19
14 → 23
15 → 16, 21
16 → 17
17 → 22
18 → 19, 22
19 → 20
20 → 23
21 → 24
22 → 31
23 → 25
24 → 26, 27
25 → 28, 29
26 → 30, 33
27 → 30, 35
28 → 32, 36
29 → 32, 38
30 → 34
31 → 40, 41
32 → 37
33 → 34, 39
34 → 35
35 → 40
36 → 37, 41
37 → 38
38 → 42
I
II 1 → 4 (i⁻¹, j)
11 → 14
39 → 42
III 1 → 39 (i, j−1)
2 → 40
3 → 41
4 → 42
IV II − (III − 1)

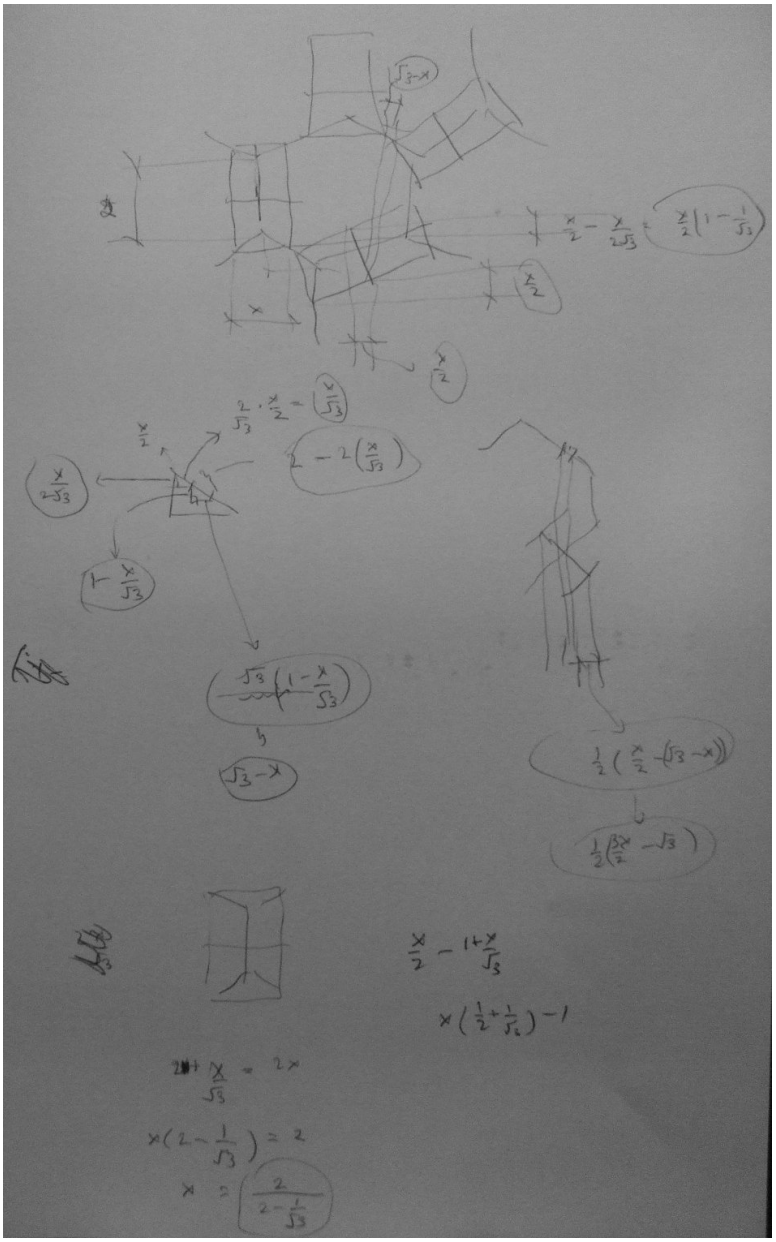


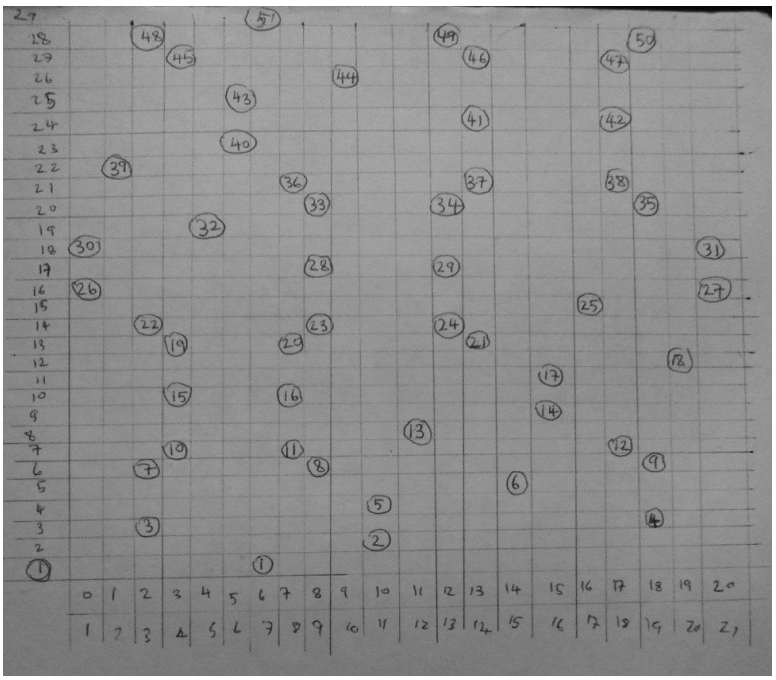

1 → 2
2 → 5
3 → 7
4 → 9
5 → 6, 8
6 → 12, 13
7 → 10
8 → 11
9 → 12
10 → 11, 15
11 → 16
12 → 14
13 → 14
14 → 17
15 → 16, 19
16 → 20
17 → 13, 21
18 → 25
19 → 22
20 → 23
21 → 24, 25
22 → 26
23 → 24, 28
24 → 29
25 → 27
26 → 30
28 → 29, 33
29 → 34
30 → 32
31 → 35
32 → 36, 39
33 → 34, 36
34 → 37
35 → 38
36 → 40
37 → 41
39 → 40
40 → 43
41 → 42, 46
42 → 47
43 → 44, 45
44 → 49
45 → 48
46 → 49
47 → 50

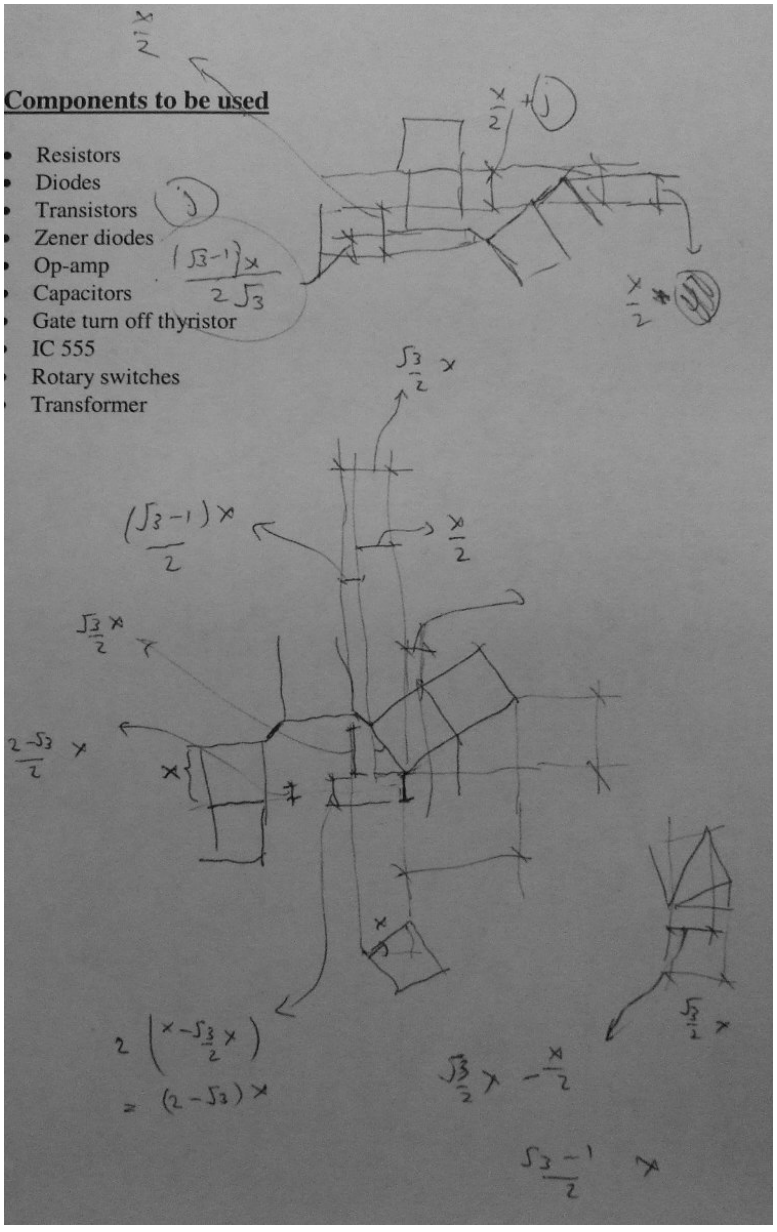


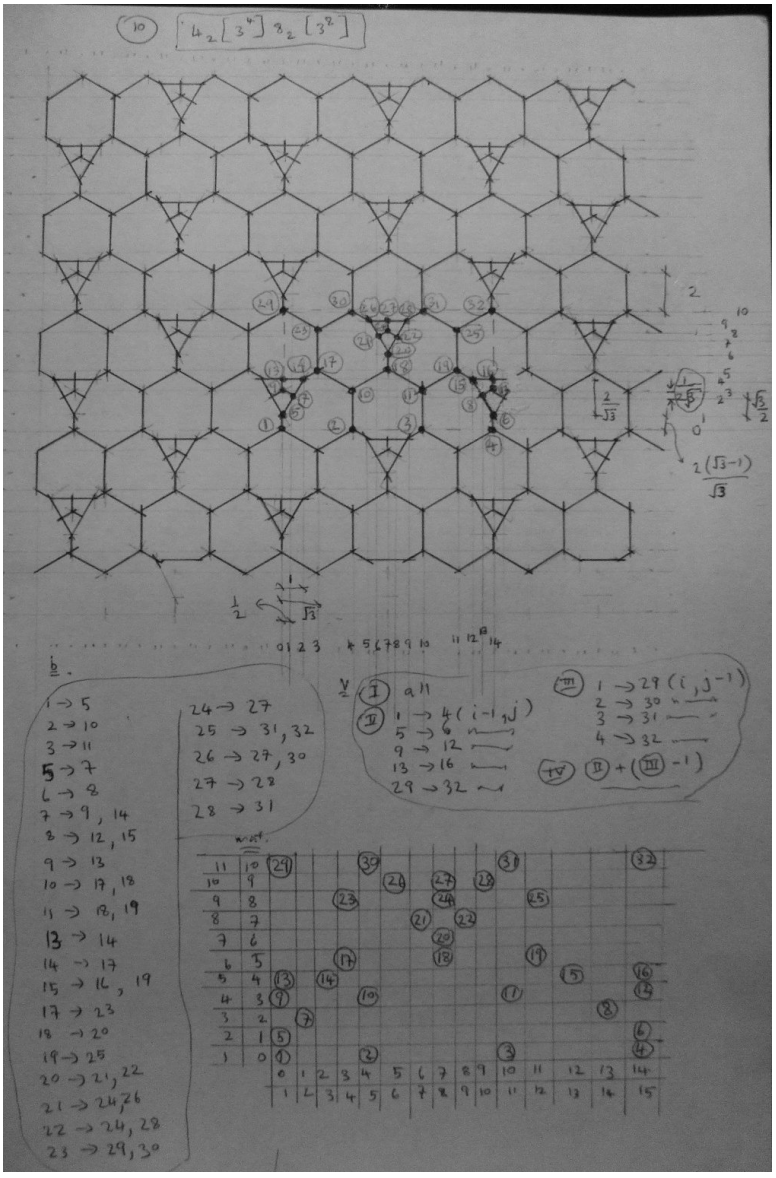

1 → 1, 7, 12, 37, 53
2 → 27
3 → 32, 35
4 → 44
5 → 2, 8, 13, 28, 54
6 → 16, 21, 25, 50
7 → 36
8 → 45, 48
9 → 5
10 → 17, 22, 26, 41
11 → 14, 28, 33, 38
12 → 49
13 → 6, 10
14 → 19
15 → 3, 30, 34, 39, 55
16 → 27, 42, 46, 51
17 → 11
18 → 20, 23
19 → 31
20 → 43, 47, 52
21 → 4, 9, 15, 40, 56
1 → 1, 2, 34
2 → 5
3 → 6
4 → 7, 8, 9
5 → 10
6 → 11
7 → 12, 13, 14, 15
8 → 16, 17, 18
9 → 19
10 → 20
11 → 21, 22
12 → 23
13 → 24
14 → 25, 26, 27
15 → 28, 29, 30
16 → 31
17 → 32
18 → 33, 34
19 → 35
20 → 36
21 → 37, 38, 39, 40
22 → 41, 42, 43
23 → 44
24 → 45
25 → 46, 47
26 → 48
27 → 49
28 → 50, 51, 52
29 → 53, 54, 55, 56
final circ → simply connected circ = {} (ϕ)

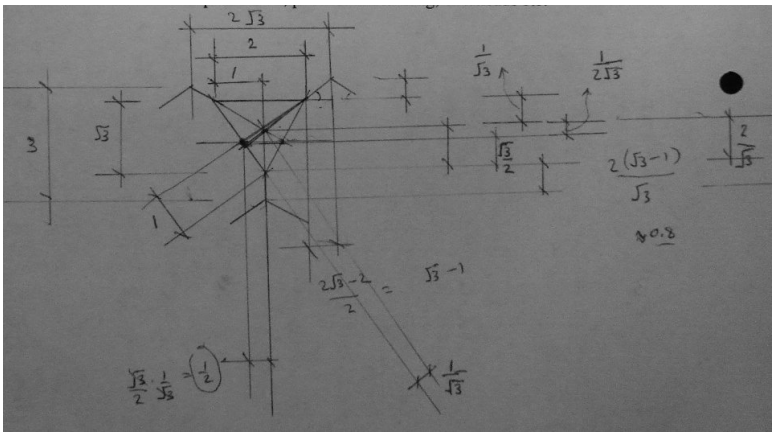


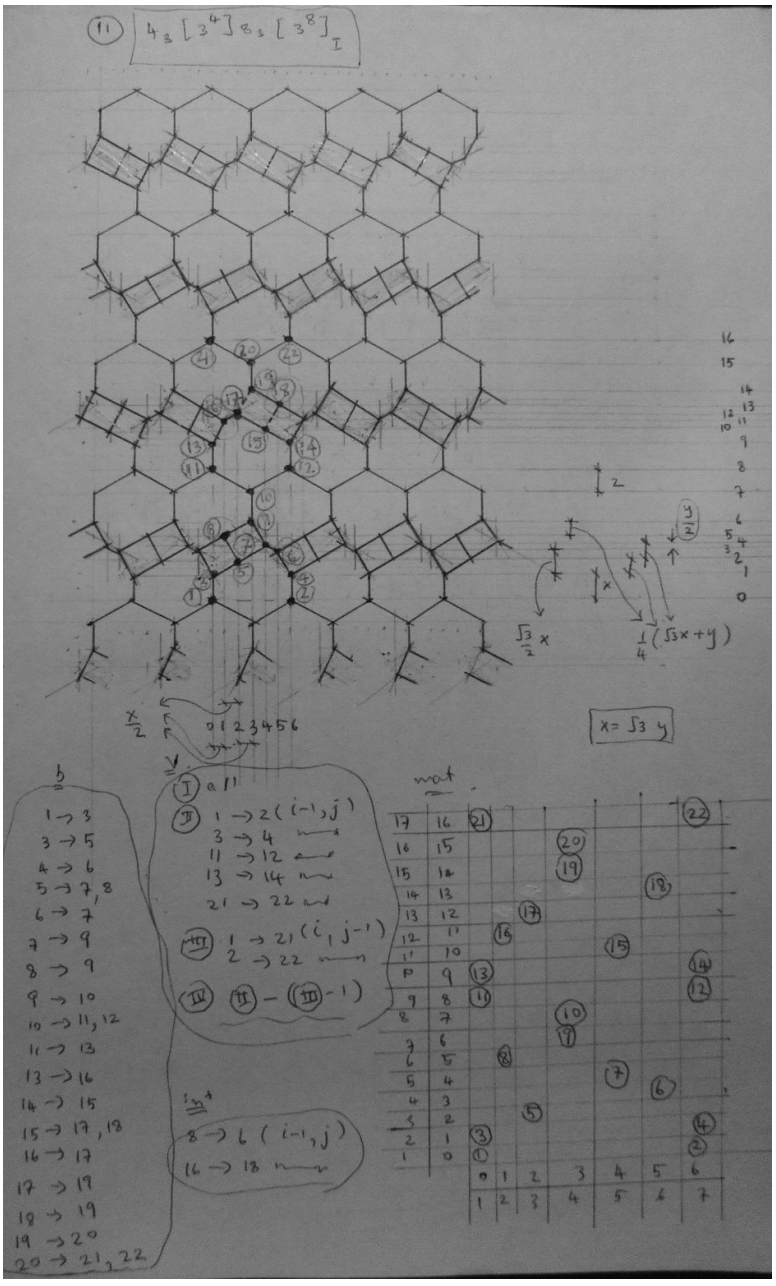


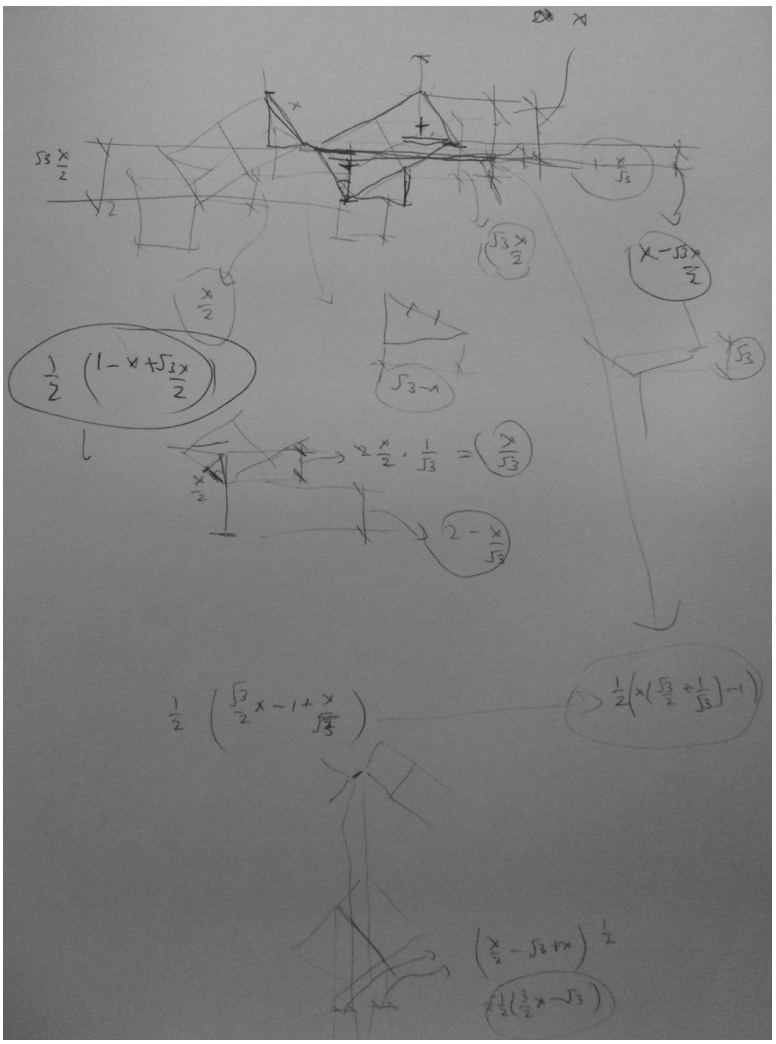


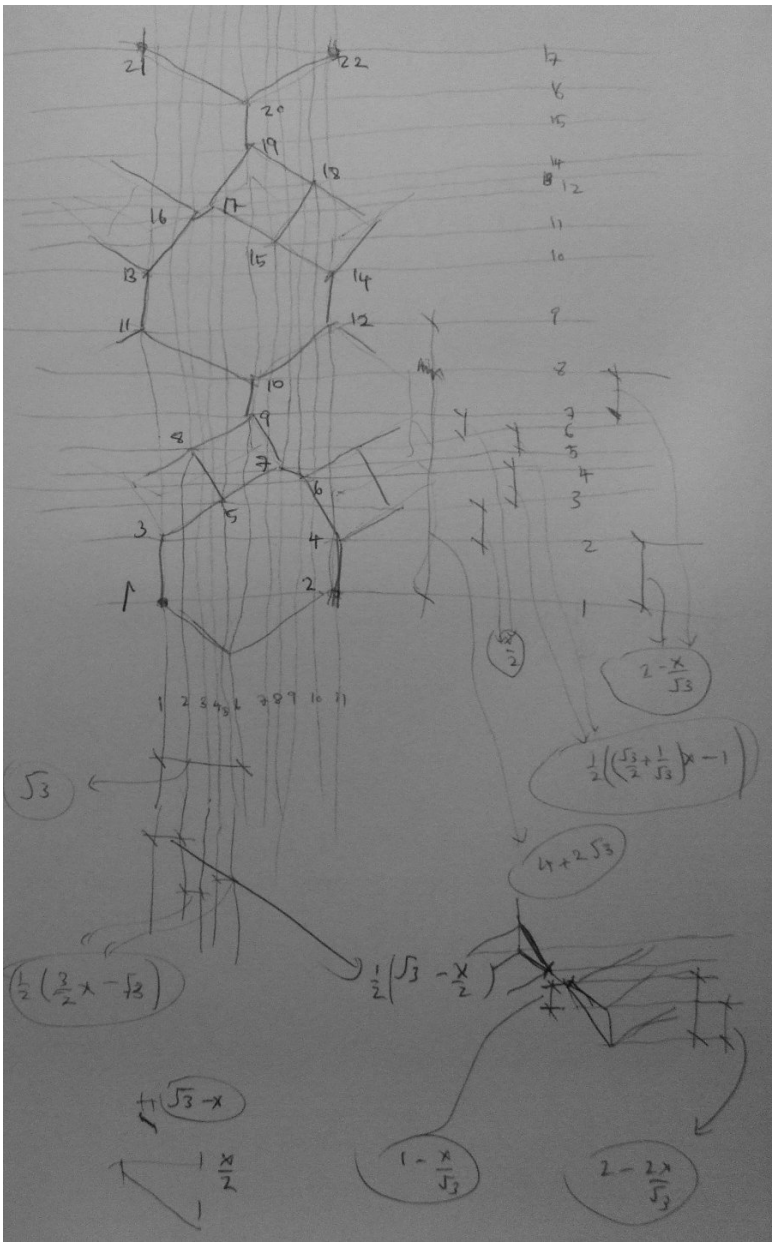

Components to be used
Resistors
Diodes
Transistors
Zener diodes
Op-amp
Capacitors
Gate turn off thyristor
IC 555
Rotary switches
Transformer

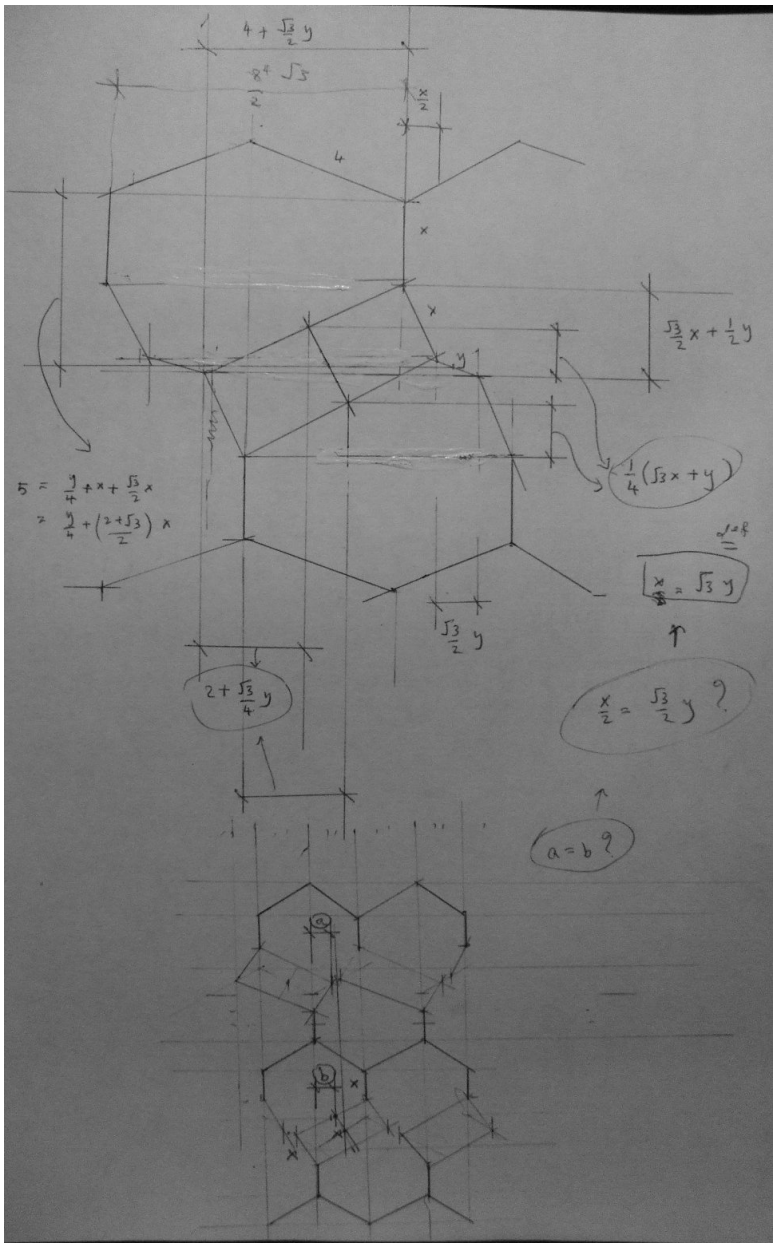

10
$4_2[3^4]\,3_2[3^2]$
2
$\frac{2}{\sqrt{3}}$
$\frac{\sqrt{3}}{2}$
$2\frac{(\sqrt{3}-1)}{\sqrt{3}}$
$\frac{1}{2}$
$\sqrt{3}$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
b.
1 → 5
2 → 10
3 → 11
5 → 7
6 → 8
7 → 9, 14
8 → 12, 15
9 → 13
10 → 17, 18
11 → 12, 19
13 → 14
14 → 17
15 → 16, 19
17 → 23
18 → 20
19 → 25
20 → 21, 22
21 → 24, 26
22 → 24, 28
23 → 29, 30
24 → 27
25 → 31, 32
26 → 27, 30
27 → 28
28 → 31
V/2
I all
II 1 → 4 (i-1, j)
5 → 8
9 → 12
13 → 16
29 → 32
III 1 → 29 (i, j-1)
2 → 30
3 → 31
4 → 32
IV II + (III -1)

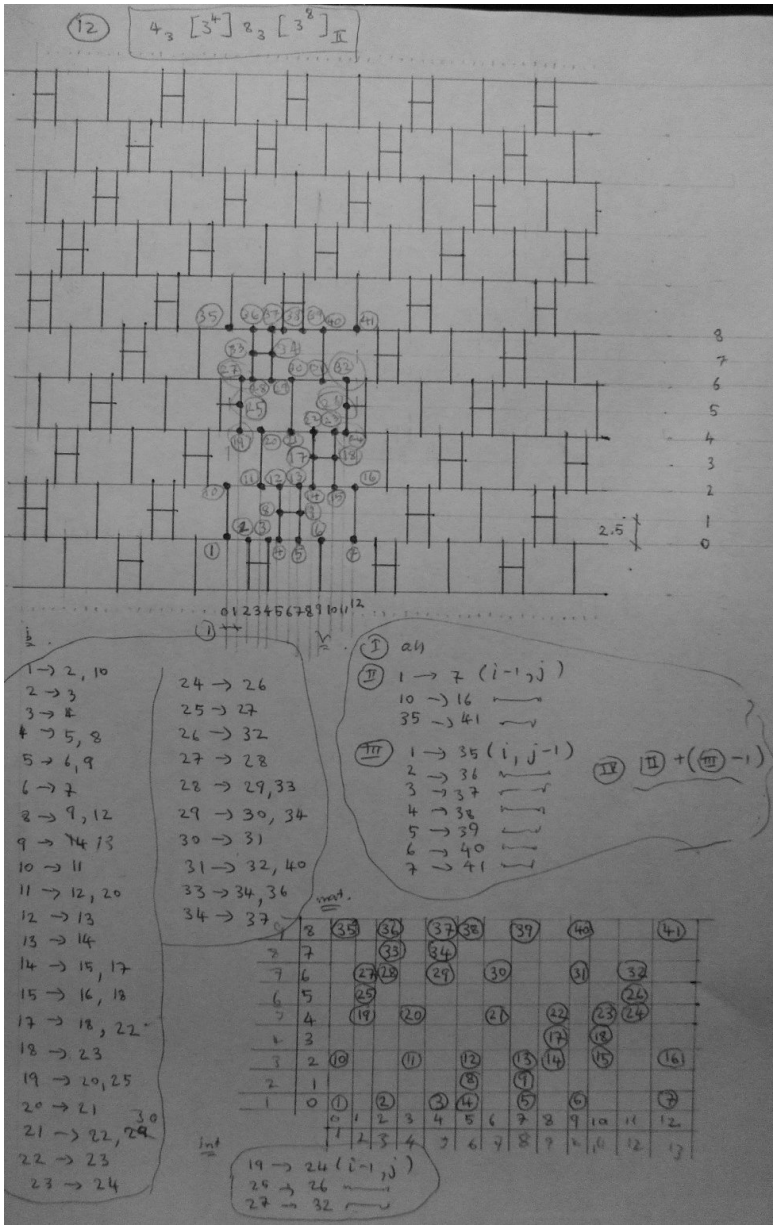

2√3
2
1
3
√3
1
√3/2
1/√3
1/2√3
2(√3-1)/√3
2/√3
2√3-2/2 = √3-1
√3/2 · 1/√3 = 1/2
1/√3

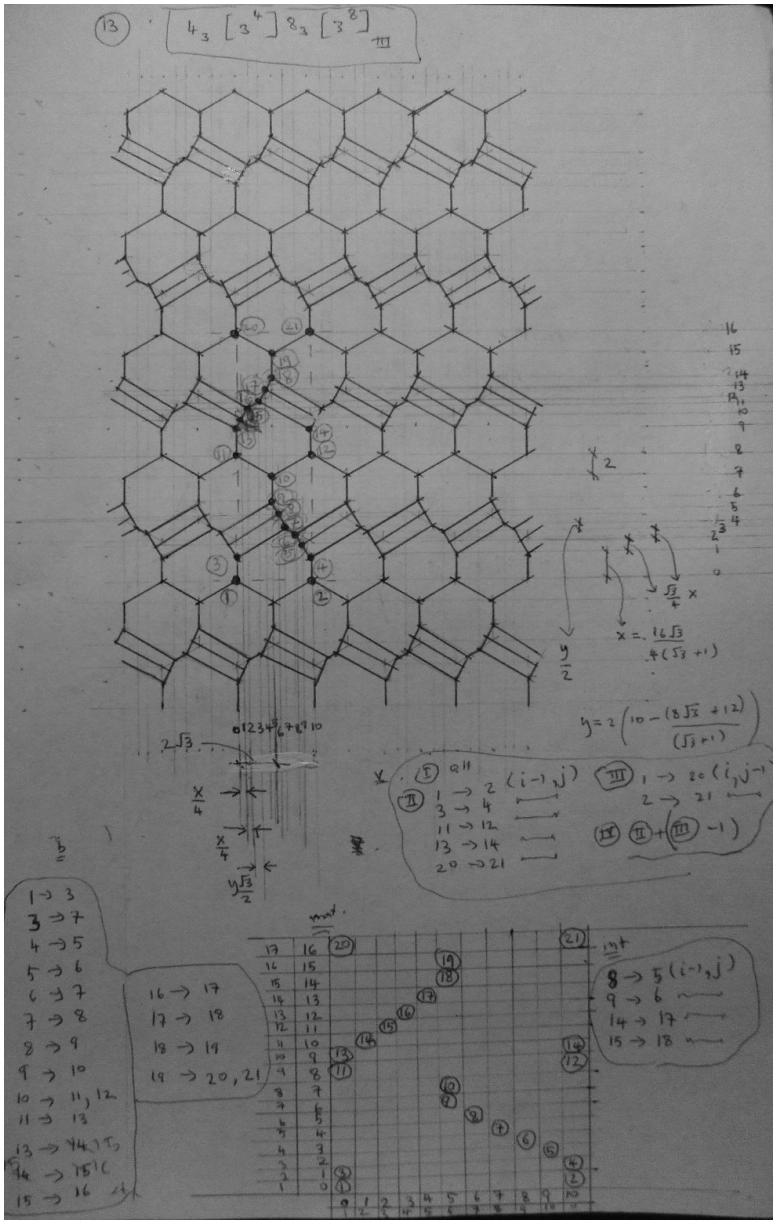

(11) $4_3[3^4]8_3[3^8]_I$
$x = \sqrt{3}\,y$
$\frac{\sqrt{3}}{2}x$
$\frac{1}{4}(\sqrt{3}x+y)$
I all
II $1 \to 2\,(i-1,j)$
$3 \to 4$
$11 \to 12$
$13 \to 14$
$21 \to 22$
III $1 \to 21\,(i, j-1)$
$2 \to 22$
IV II − (III − 1)
$8 \to 6\,(i-1, j)$
$16 \to 18$

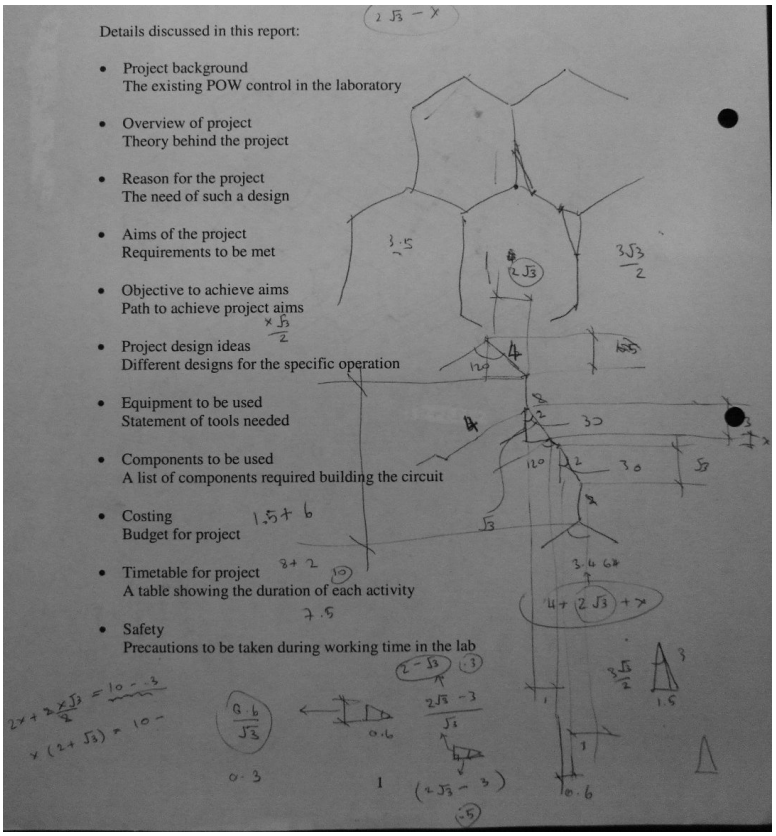


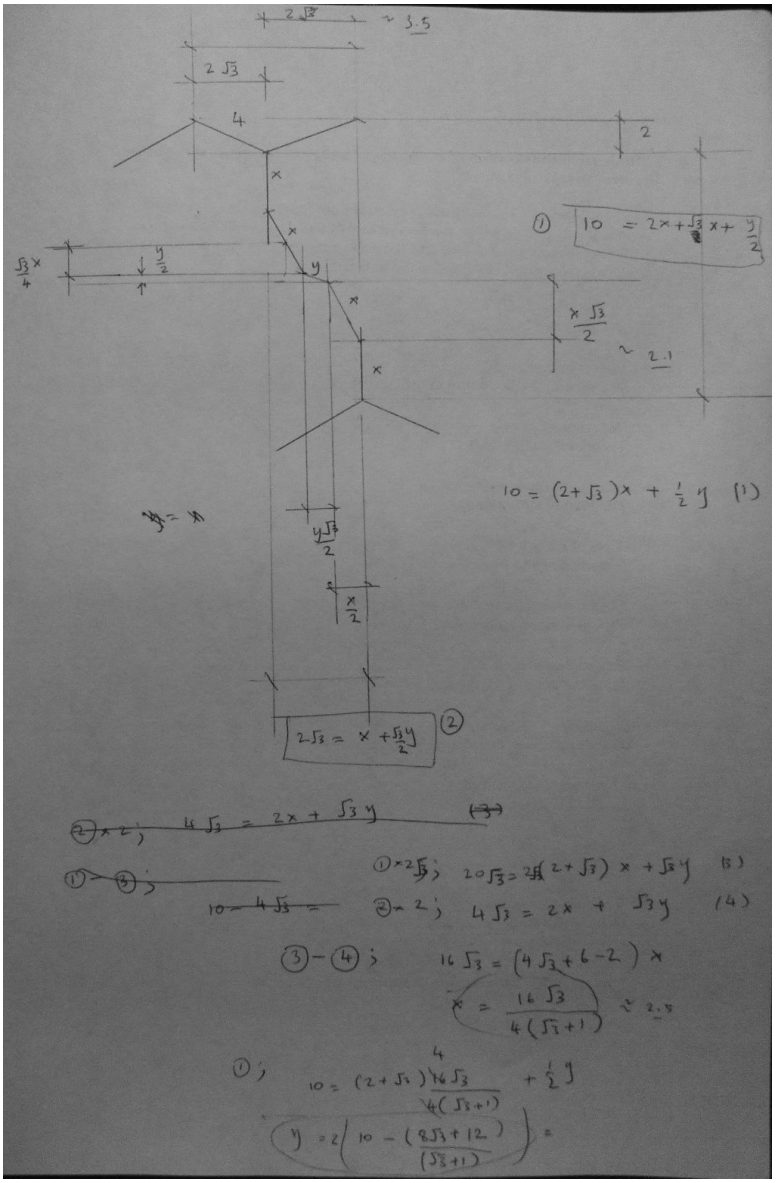

21
22
20
19
18
16
17
15
13
14
11
12
10
9
8
7
6
5
3
4
2
1
$\frac{\sqrt{3}}{2}$
$2-\frac{x}{\sqrt{3}}$
$\frac{1}{2}\left(\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}\right)x-1\right)$
$\sqrt{3}$
$4+2\sqrt{3}$
$\frac{1}{2}\left(\frac{3}{2}x-\sqrt{3}\right)$
$\frac{1}{2}\left(\sqrt{3}-\frac{x}{2}\right)$
$\sqrt{3}-x$
$\frac{x}{2}$
$1-\frac{x}{\sqrt{3}}$
$2-\frac{2x}{\sqrt{3}}$

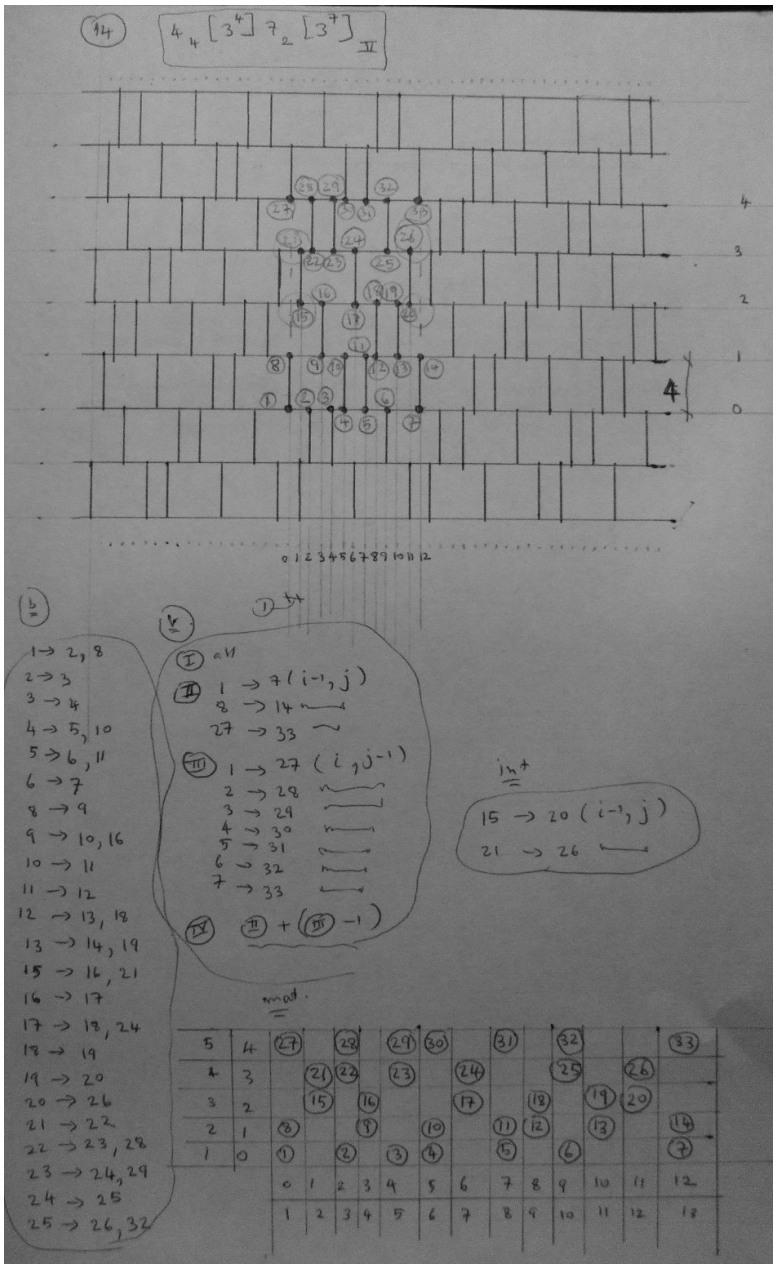

4 + √3/2 y
8^q √3/2
x/2
4
x
y
√3/2 x + 1/2 y
1/4 (√3 x + y)
5 = y/4 + x + √3/2 x
= y/4 + (2+√3)/2 x
x/2 = √3/2 y
√3/2 y
2 + √3/4 y
x/2 = √3/2 y ?
a = b ?
a
b

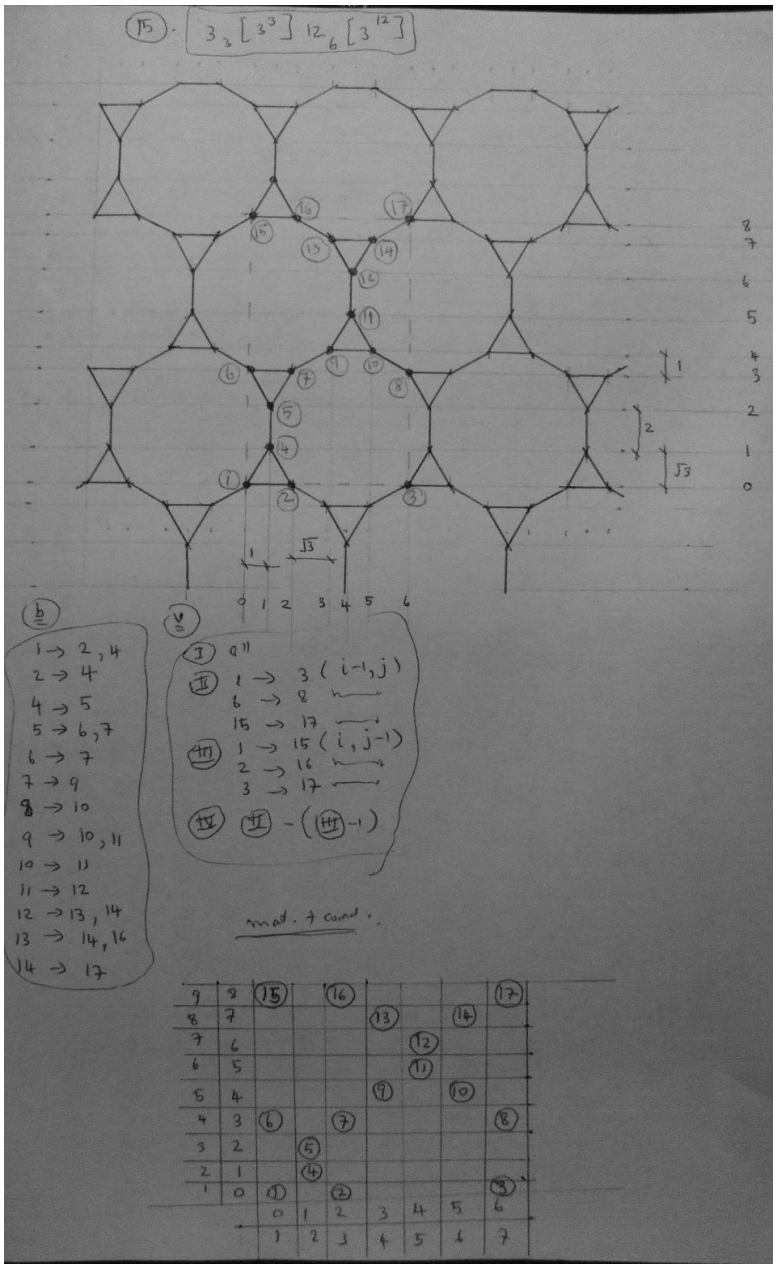

12
$4_3\,[3^4]\,8_3\,[3^8]$ II
2.5
0 1 2 3 4 5 6 7 8 9 10 11 12
1 → 2, 10
2 → 3
3 → 8
4 → 5, 8
5 → 6, 9
6 → 7
8 → 9, 12
9 → 14 13
10 → 11
11 → 12, 20
12 → 13
13 → 14
14 → 15, 17
15 → 16, 18
17 → 18, 22
18 → 23
19 → 20, 25
20 → 21
21 → 22, 29 30
22 → 23
23 → 24
24 → 26
25 → 27
26 → 32
27 → 28
28 → 29, 33
29 → 30, 34
30 → 31
31 → 32, 40
33 → 34, 36
34 → 37
I all
II 1 → 7 (i-1, j)
10 → 16
35 → 41
III 1 → 35 (i, j-1)
2 → 36
3 → 37
4 → 38
5 → 39
6 → 40
7 → 41
IV II + (III -1)
19 → 24 (i-1, j)
25 → 26
27 → 32

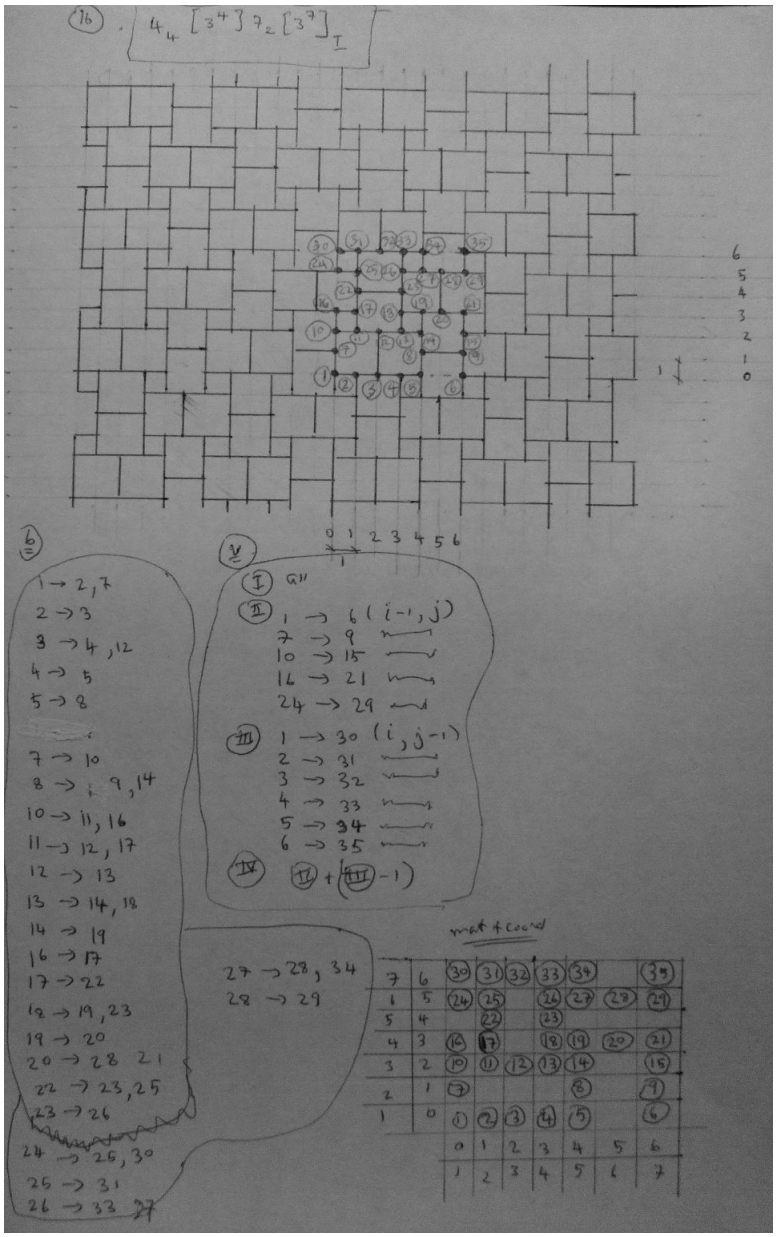

13
$4_3\ [3^4]\ 8_3\ [3^3.8]$
III
$\frac{y}{2}$
$\frac{\sqrt{3}}{4}x$
$x = \frac{16\sqrt{3}}{4(\sqrt{3}+1)}$
$y = 2\left(10 - \frac{(8\sqrt{3}+12)}{(\sqrt{3}+1)}\right)$
$2\sqrt{3}$
$\frac{x}{4}$
$\frac{y\sqrt{3}}{2}$
1 → 3
3 → 7
4 → 5
5 → 6
6 → 7
7 → 8
8 → 9
9 → 10
10 → 11, 12
11 → 13
13 → 14, 15
14 → 15, 16
15 → 16
16 → 17
17 → 18
18 → 19
19 → 20, 21
8 → 5 (i-1, j)
9 → 6
14 → 17
15 → 18

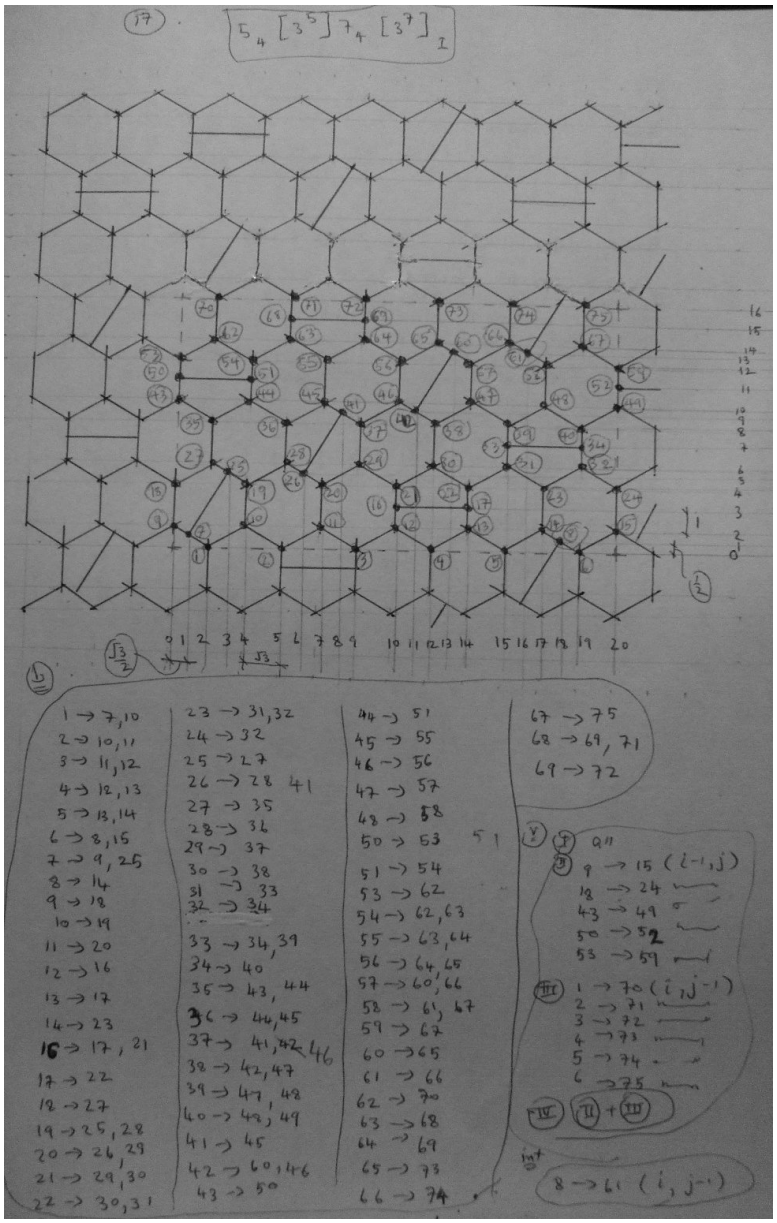

Details discussed in this report:
Project background
The existing POW control in the laboratory
Overview of project
Theory behind the project
Reason for the project
The need of such a design
Aims of the project
Requirements to be met
Objective to achieve aims
Path to achieve project aims
Project design ideas
Different designs for the specific operation
Equipment to be used
Statement of tools needed
Components to be used
A list of components required building the circuit
Costing
Budget for project
Timetable for project
A table showing the duration of each activity
Safety
Precautions to be taken during working time in the lab

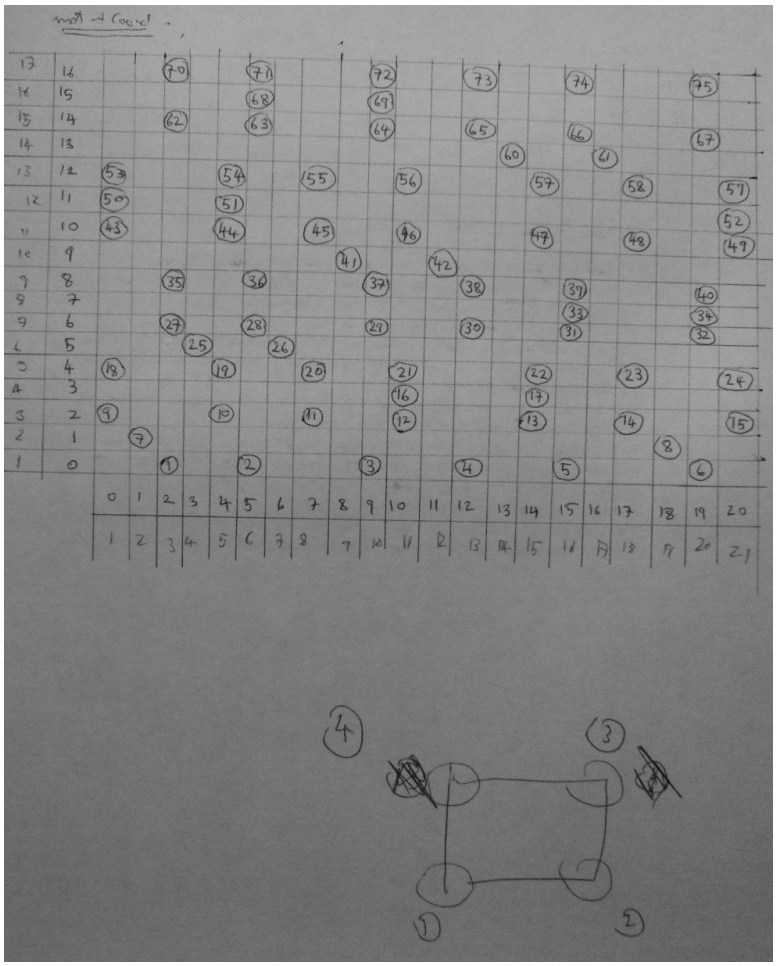

2√3
≈ 3.5
2√3
4
2
x
x
y/2
√3/4 x
y
x
x
x√3/2
≈ 2.1
x
① 10 = 2x + √3 x + y/2
10 = (2+√3)x + ½y (1)
y = x
4√3/2
x/2
2√3 = x + √3/2 y ②
② × 2; 4√3 = 2x + √3 y (3)
① − ③; 10 − 4√3 = (2−√3)x + ½y (5)
① × 2√3; 20√3 = 2√3(2+√3)x + √3 y (5)
③ − 2; 4√3 = 2x + √3 y (4)
③ − ④; 16√3 = (4√3 + 6 − 2)x
x = 16√3 / 4(√3+1) ≈ 2.5
①; 10 = (2+√3)·16√3/4(√3+1) + ½y
y = 2(10 − (8√3+12)/(√3+1))

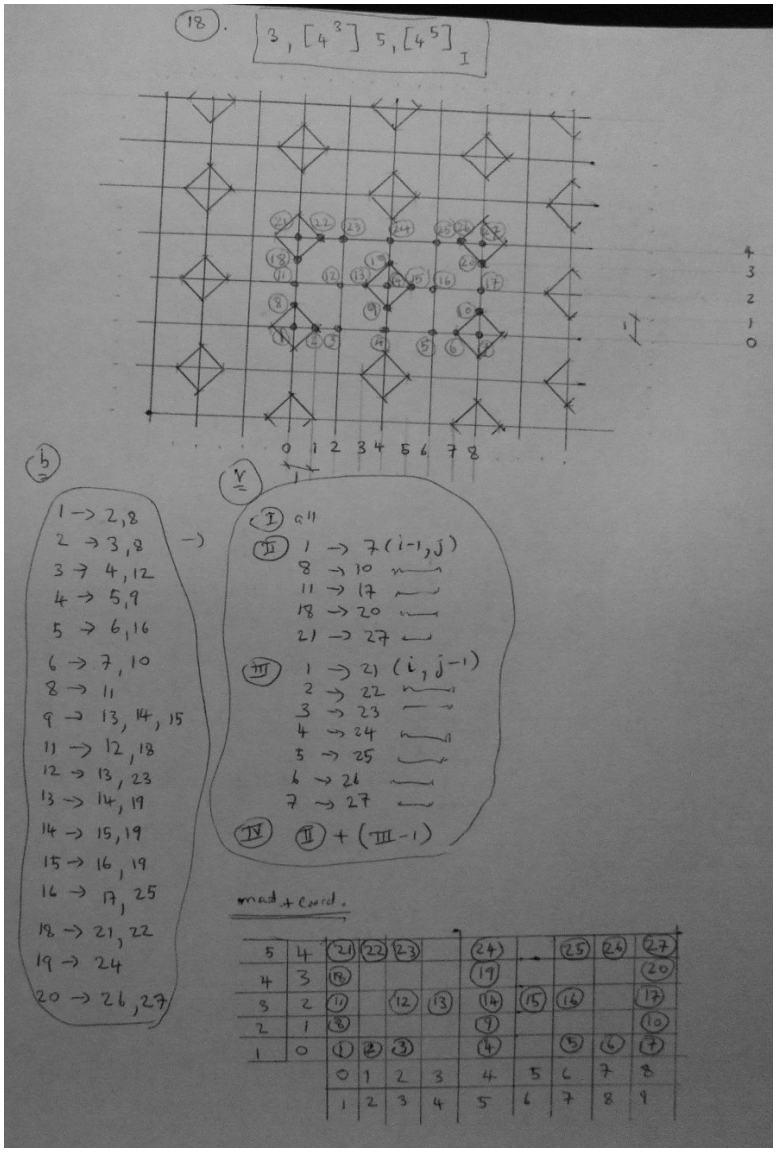

14
$4_4[3^4]7_2[3^7]$ II
4
0 1 2 3 4 5 6 7 8 9 10 11 12
1 → 2, 8
2 → 3
3 → 4
4 → 5, 10
5 → 6, 11
6 → 7
8 → 9
9 → 10, 16
10 → 11
11 → 12
12 → 13, 18
13 → 14, 19
15 → 16, 21
16 → 17
17 → 18, 24
18 → 19
19 → 20
20 → 26
21 → 22
22 → 23, 28
23 → 24, 29
24 → 25
25 → 26, 32
I all
II 1 → 7 (i-1, j)
8 → 14
27 → 33
III 1 → 27 (i, j-1)
2 → 28
3 → 29
4 → 30
5 → 31
6 → 32
7 → 33
IV II + (III −1)
mod
int
15 → 20 (i-1, j)
21 → 26

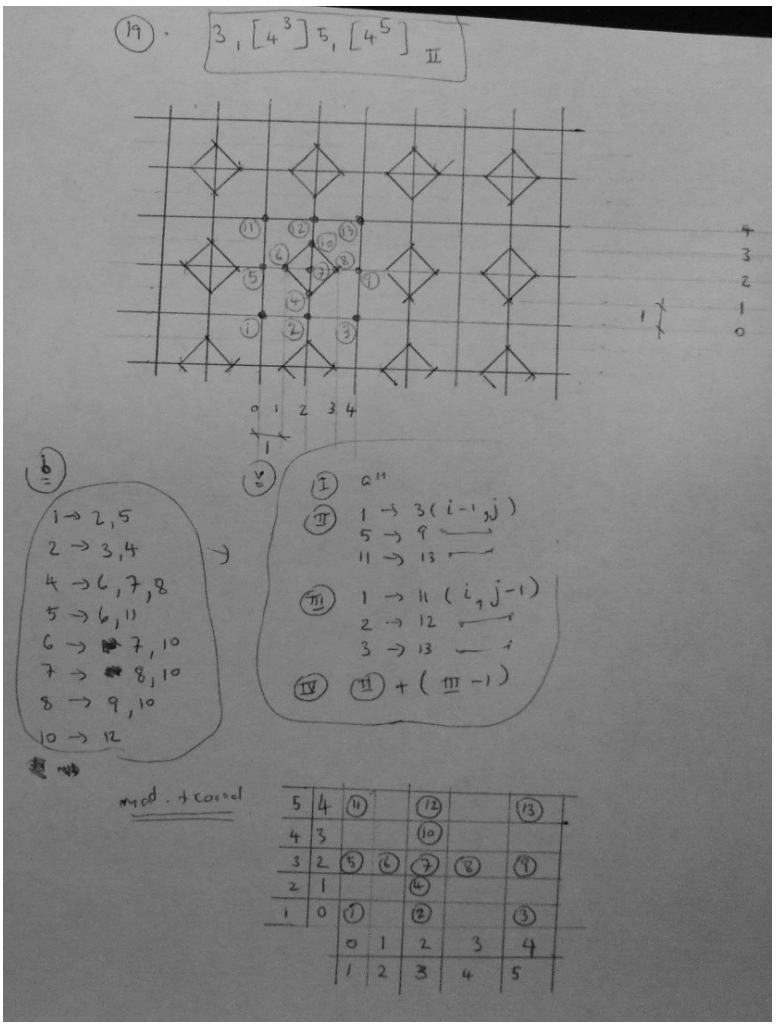

15
$3_3[3^3]\ 12_6[3^{12}]$
1
$\sqrt{3}$
2
$\sqrt{3}$
b
v
I
II
III
IV
$i \to 2, 4$
$2 \to 4$
$4 \to 5$
$5 \to 6, 7$
$6 \to 7$
$7 \to 9$
$8 \to 10$
$9 \to 10, 11$
$10 \to 11$
$11 \to 12$
$12 \to 13, 14$
$13 \to 14, 16$
$14 \to 17$
$1 \to 3\ (i-1, j)$
$6 \to 8$
$15 \to 17$
$1 \to 15\ (i, j-1)$
$2 \to 16$
$3 \to 17$
IV − (III − 1)

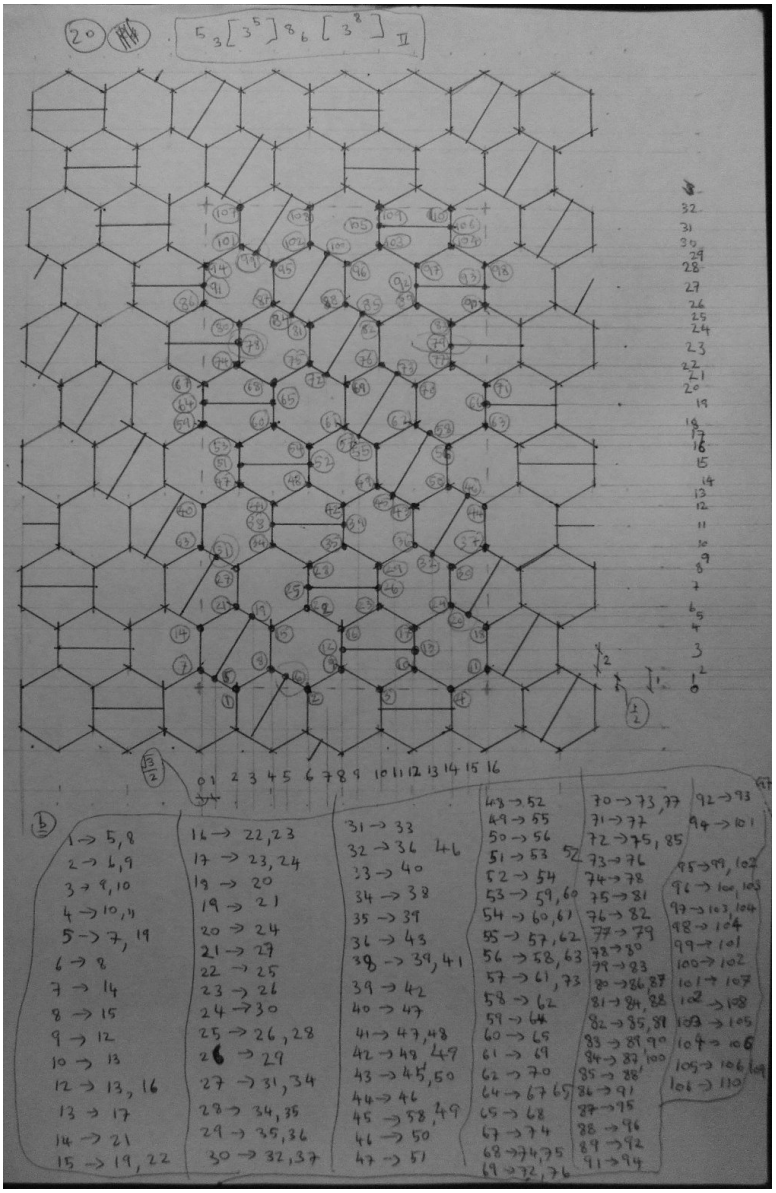

16
$4_4\,[3^4]\,7_2\,[3^7]_I$
6 5 4 3 2 1 0
0 1 2 3 4 5 6
6
V
I
II
1 → 6 (i-1, j)
7 → 9
10 → 15
16 → 21
24 → 29
III
1 → 30 (i, j-1)
2 → 31
3 → 32
4 → 33
5 → 34
6 → 35
IV
II + (III - 1)
1 → 2, 7
2 → 3
3 → 4, 12
4 → 5
5 → 8
7 → 10
8 → 9, 14
10 → 11, 16
11 → 12, 17
12 → 13
13 → 14, 18
14 → 19
16 → 17
17 → 22
18 → 19, 23
19 → 20
20 → 28 21
22 → 23, 25
23 → 26
24 → 26, 30
25 → 31
26 → 33
27 → 28, 34
28 → 29
mat + coord

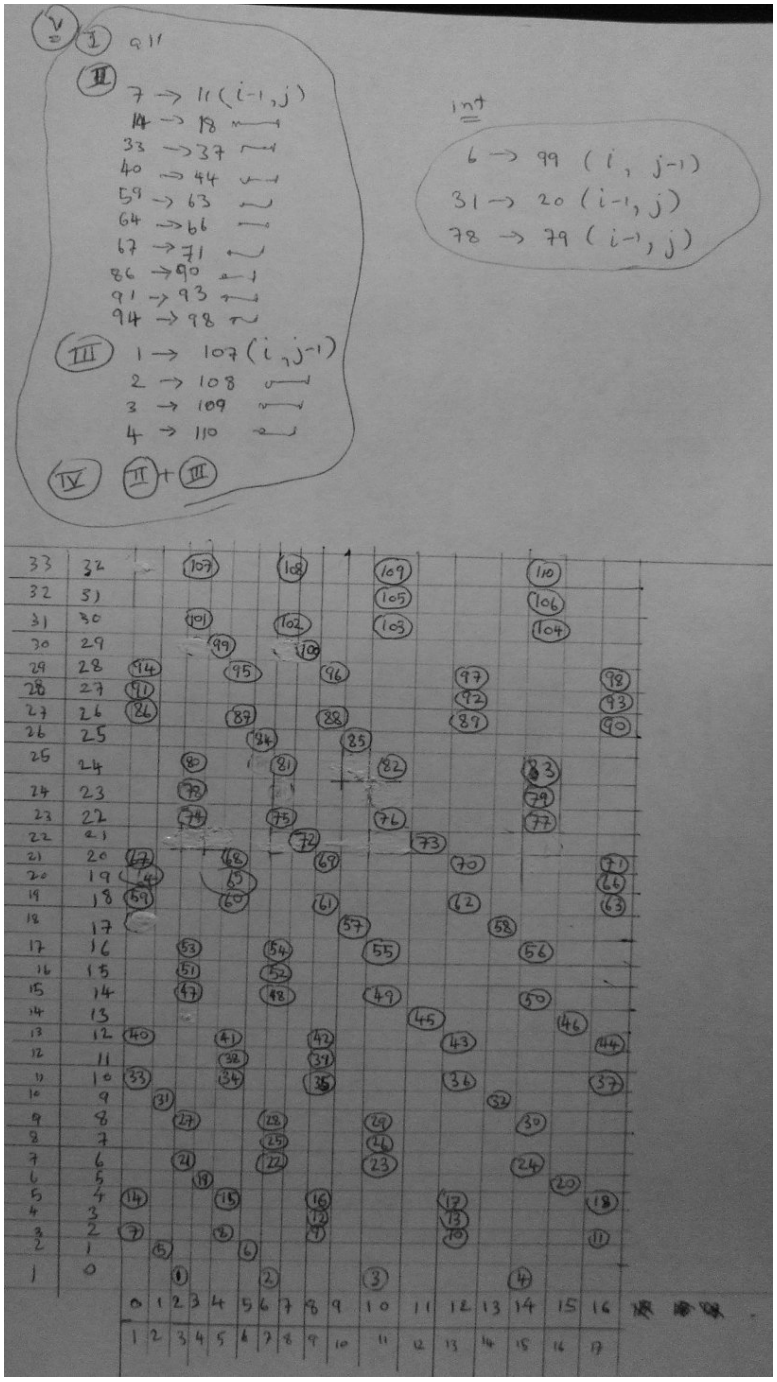

17
$5_4\,[3^5]\,7_4\,[3^7]_I$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
1 → 7,10
2 → 10,11
3 → 11,12
4 → 12,13
5 → 13,14
6 → 8,15
7 → 9,25
8 → 14
9 → 18
10 → 19
11 → 20
12 → 16
13 → 17
14 → 23
16 → 17, 21
17 → 22
18 → 27
19 → 25,28
20 → 26,29
21 → 29,30
22 → 30,31
23 → 31,32
24 → 32
25 → 27
26 → 28 41
27 → 35
28 → 36
29 → 37
30 → 38
31 → 33
32 → 34
33 → 34,39
34 → 40
35 → 43, 44
36 → 46,45
37 → 41,42,46
38 → 42,47
39 → 47, 48
40 → 48, 49
41 → 45
42 → 60,46
43 → 50
44 → 51
45 → 55
46 → 56
47 → 57
48 → 58
50 → 53
51 → 54
53 → 62
54 → 62,63
55 → 63,64
56 → 64,65
57 → 60,66
58 → 61, 67
59 → 67
60 → 65
61 → 66
62 → 70
63 → 68
64 → 69
65 → 73
66 → 74
67 → 75
68 → 69, 71
69 → 72
9 → 15 (i-1, j)
18 → 24
43 → 49
50 → 52
53 → 59
1 → 70 (i, j-1)
2 → 71
3 → 72
4 → 73
5 → 74
6 → 75
8 → 61 (i, j-1)

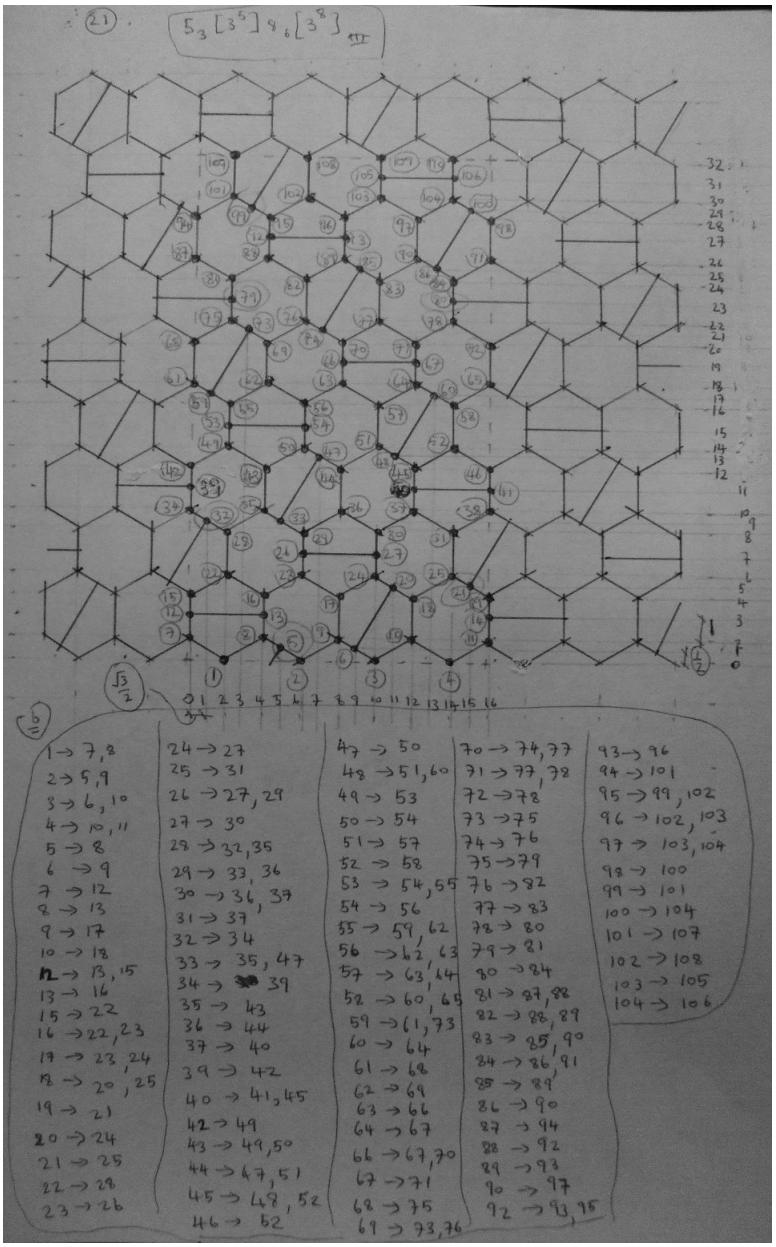


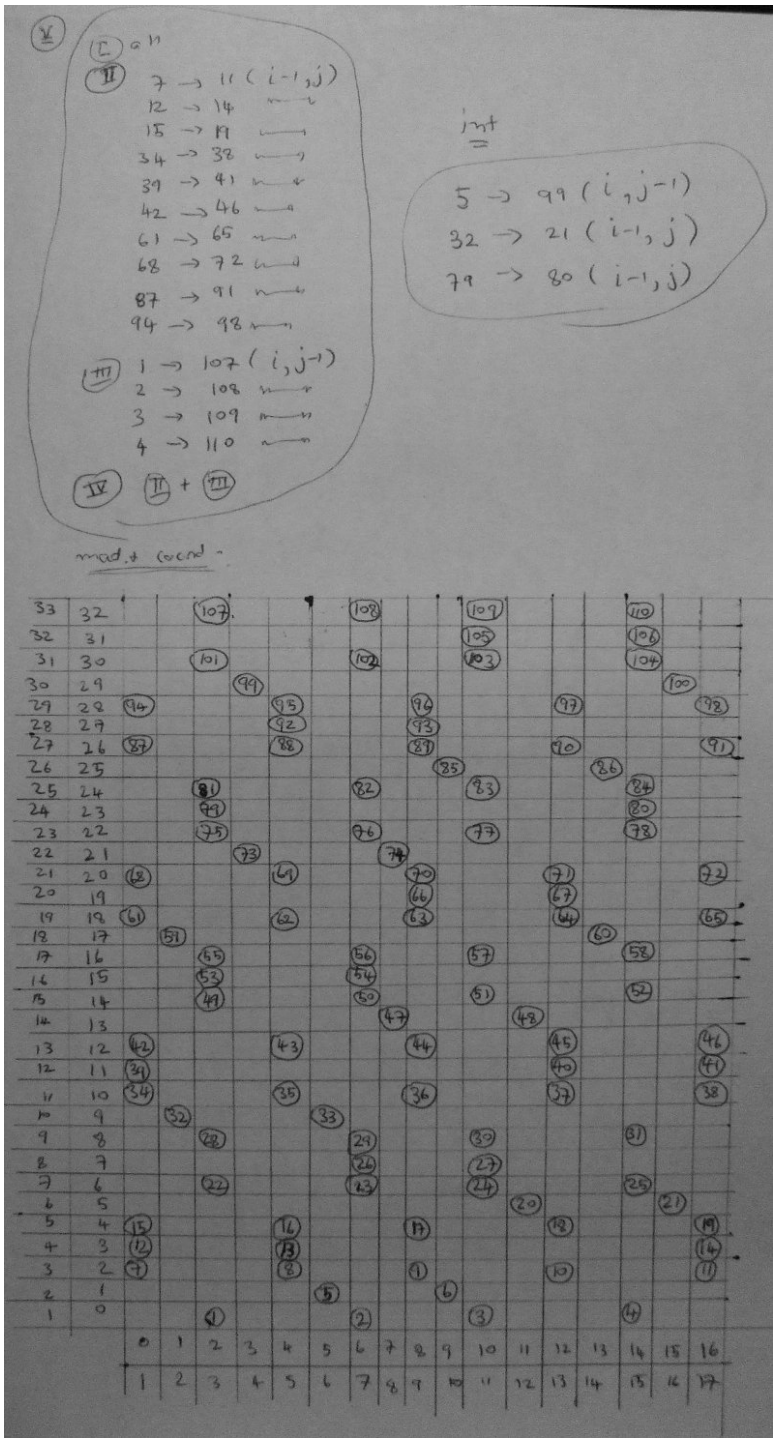

18.
3, [4^3] 5, [4^5]
I
0 1 2 3 4 5 6 7 8
4 3 2 1 0
b
1 -> 2,8
2 -> 3,8
3 -> 4,12
4 -> 5,9
5 -> 6,16
6 -> 7,10
8 -> 11
9 -> 13, 14, 15
11 -> 12, 18
12 -> 13, 23
13 -> 14, 19
14 -> 15, 19
15 -> 16, 19
16 -> 17, 25
18 -> 21, 22
19 -> 24
20 -> 26, 27
V
I all
II 1 -> 7 (i-1, j)
8 -> 10
11 -> 17
18 -> 20
21 -> 27
III 1 -> 21 (i, j-1)
2 -> 22
3 -> 23
4 -> 24
5 -> 25
6 -> 26
7 -> 27
IV II + (III - 1)
mat + coord.

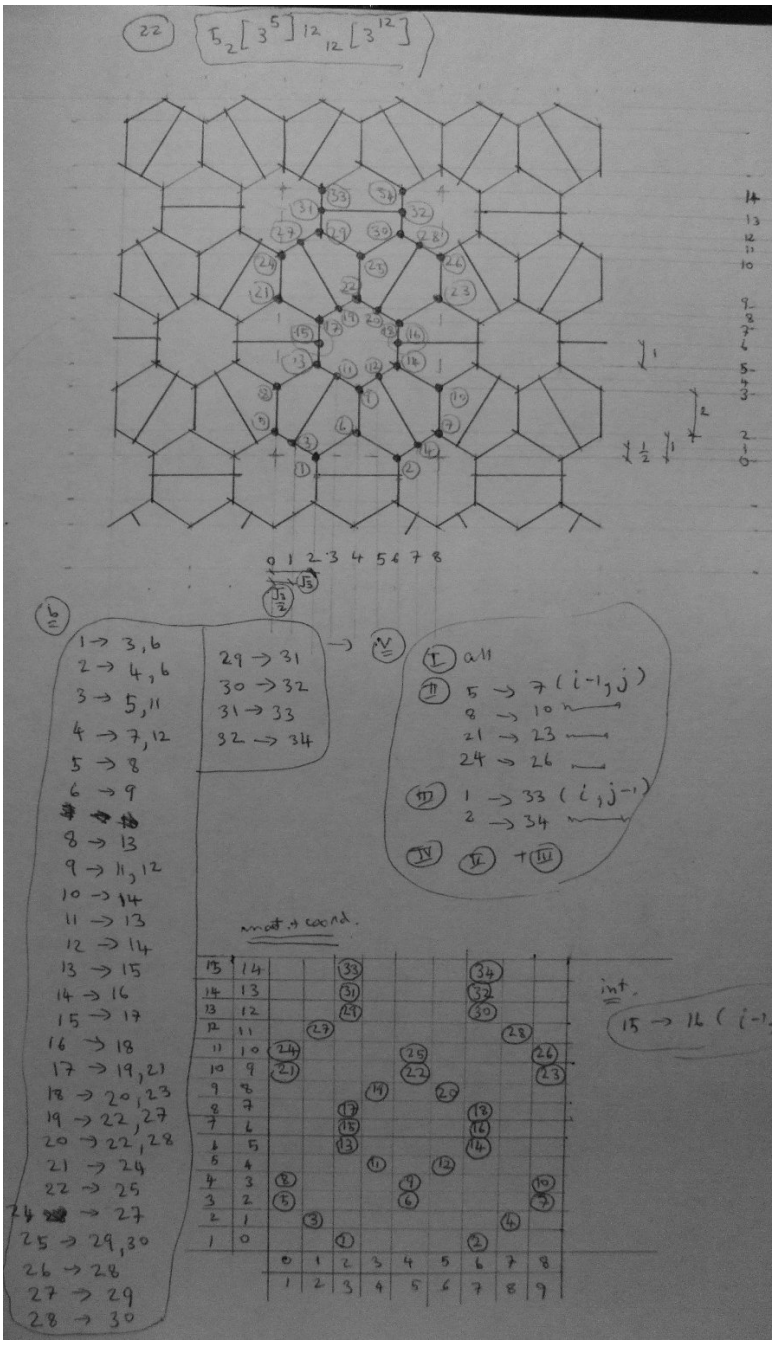

19.
3, [4^3] 5, [4^5] II
0 1 2 3 4
4 3 2 1 0
1 → 2,5
2 → 3,4
4 → 6,7,8
5 → 6,11
6 → 7,10
7 → 8,10
8 → 9,10
10 → 12
I a''
II 1 → 3(i-1,j)
5 → 9
11 → 13
III 1 → 11 (i, j-1)
2 → 12
3 → 13
IV II + (III -1)
mod. + coord

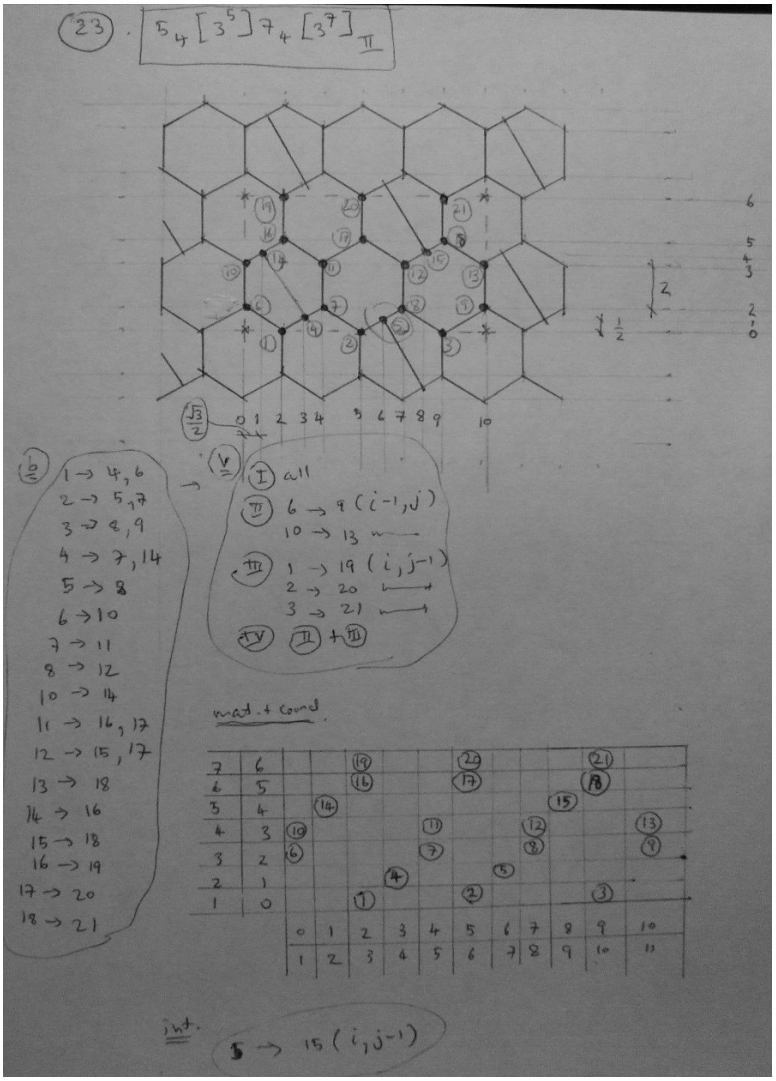

20
$5_3[3^5]8_6[3^8]$ II
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 → 5, 8
2 → 4, 9
3 → 9, 10
4 → 10, 11
5 → 7, 19
6 → 8
7 → 14
8 → 15
9 → 12
10 → 13
12 → 13, 16
13 → 17
14 → 21
15 → 19, 22
16 → 22, 23
17 → 23, 24
19 → 20
20 → 24
21 → 27
22 → 25
23 → 26
24 → 30
25 → 26, 28
26 → 29
27 → 31, 34
28 → 34, 35
29 → 35, 36
30 → 32, 37
31 → 33
32 → 36, 46
33 → 40
34 → 38
35 → 39
36 → 43
38 → 39, 41
39 → 42
40 → 47
41 → 47, 48
42 → 48, 49
43 → 45, 50
44 → 46
45 → 58, 49
46 → 50
47 → 51
43 → 52
49 → 55
50 → 56
51 → 53, 52
52 → 54
53 → 59, 60
54 → 60, 61
55 → 57, 62
56 → 58, 63
57 → 61, 73
58 → 62
59 → 64
60 → 65
61 → 69
62 → 70
64 → 67, 65
65 → 68
67 → 74
68 → 74, 75
69 → 72, 76
70 → 73, 77
71 → 77
72 → 75, 85
73 → 76
74 → 78
75 → 81
76 → 82
77 → 79
78 → 80
79 → 83
80 → 86, 87
81 → 84, 88
82 → 85, 81
83 → 89, 90
84 → 87, 100
85 → 88
86 → 91
87 → 75
88 → 96
89 → 92
91 → 94
92 → 93
94 → 101
95 → 99, 102
96 → 100, 103
97 → 103, 104
98 → 104
99 → 101
100 → 102
101 → 107
102 → 108
103 → 105
104 → 106
105 → 106, 109
106 → 110

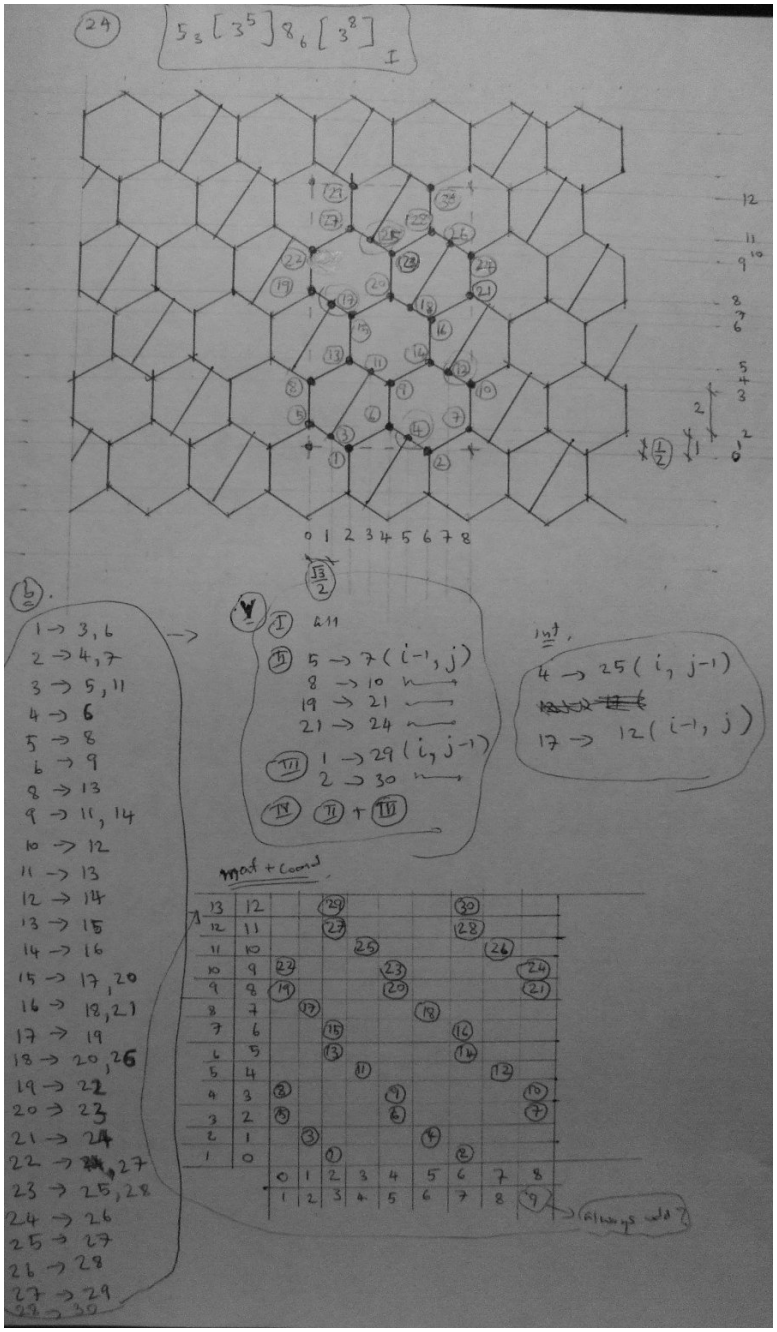

V
I
II
7 → 11 (i-1, j)
14 → 18
33 → 37
40 → 44
59 → 63
64 → 66
67 → 71
86 → 90
91 → 93
94 → 98
III
1 → 107 (i, j-1)
2 → 108
3 → 109
4 → 110
IV
II + III
int
6 → 99 (i, j-1)
31 → 20 (i-1, j)
78 → 79 (i-1, j)

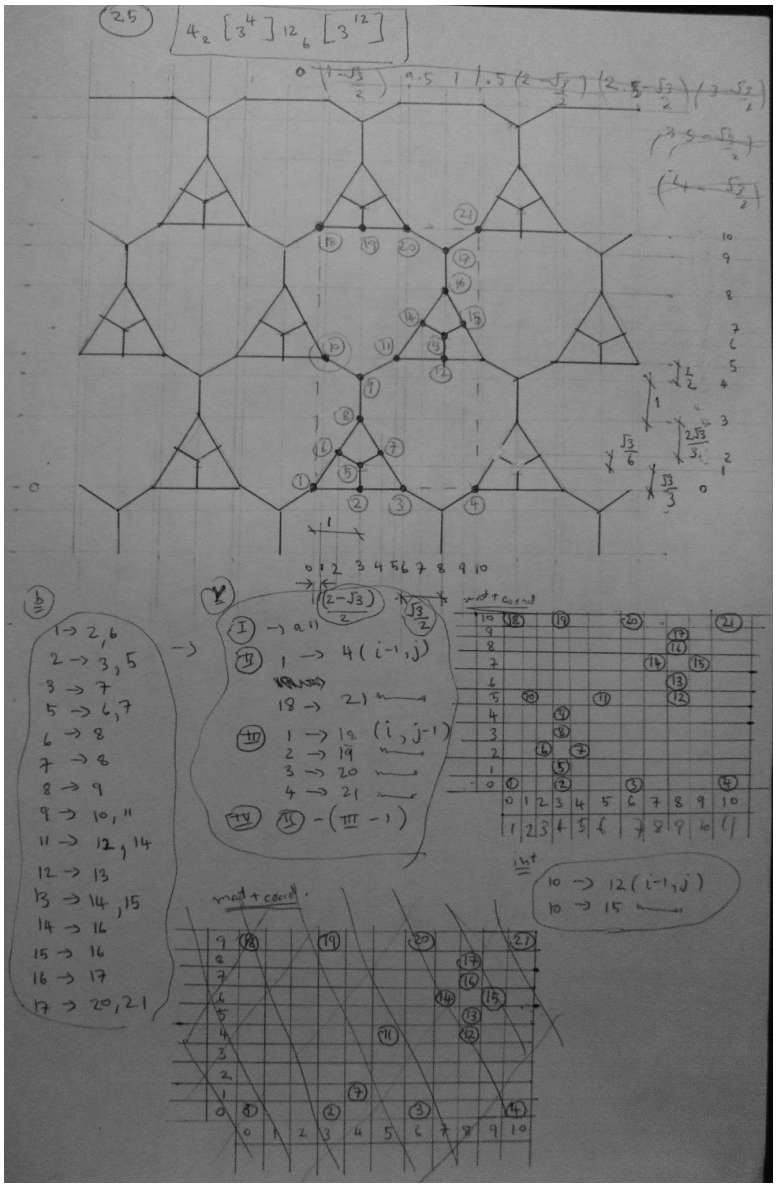

21
$5_3[3^5]8_6[3^8]$
1→ 7,8
2→ 5,9
3→ 6,10
4→ 10,11
5→ 8
6→ 9
7→ 12
8→ 13
9→ 17
10→ 18
11→ 13,15
13→ 16
15→ 22
16→ 22,23
17→ 23,24
18→ 20,25
19→ 21
20→ 24
21→ 25
22→ 28
23→ 26
24→ 27
25→ 31
26→ 27,29
27→ 30
28→ 32,35
29→ 33,36
30→ 36,39
31→ 37
32→ 34
33→ 35,47
34→ 39
35→ 43
36→ 44
37→ 40
39→ 42
40→ 41,45
42→ 49
43→ 49,50
44→ 47,51
45→ 48,52
46→ 52
47→ 50
48→ 51,60
49→ 53
50→ 54
51→ 57
52→ 58
53→ 54,55
54→ 56
55→ 59,62
56→ 62,63
57→ 63,64
58→ 60,65
59→ 61,73
60→ 64
61→ 68
62→ 69
63→ 66
64→ 67
66→ 67,70
67→ 71
68→ 75
69→ 73,76
70→ 74,77
71→ 77,78
72→ 78
73→ 75
74→ 76
75→ 79
76→ 82
77→ 83
78→ 80
79→ 81
80→ 84
81→ 87,88
82→ 88,89
83→ 85,90
84→ 86,91
85→ 89
86→ 90
87→ 94
88→ 92
89→ 93
90→ 97
92→ 93,95
93→ 96
94→ 101
95→ 99,102
96→ 102,103
97→ 103,104
98→ 100
99→ 101
100→ 104
101→ 107
102→ 108
103→ 105
104→ 106

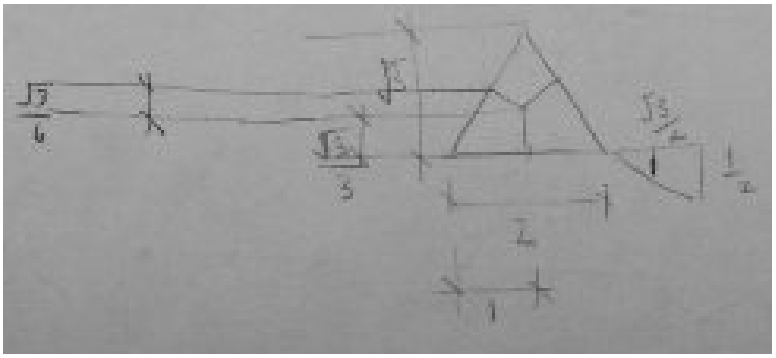

7 → 11 (i-1, j)
12 → 14
15 → 19
34 → 38
39 → 41
42 → 46
61 → 65
68 → 72
87 → 91
94 → 98
1 → 107 (i, j-1)
2 → 108
3 → 109
4 → 110
II + III
5 → 99 (i, j-1)
32 → 21 (i-1, j)
79 → 80 (i-1, j)

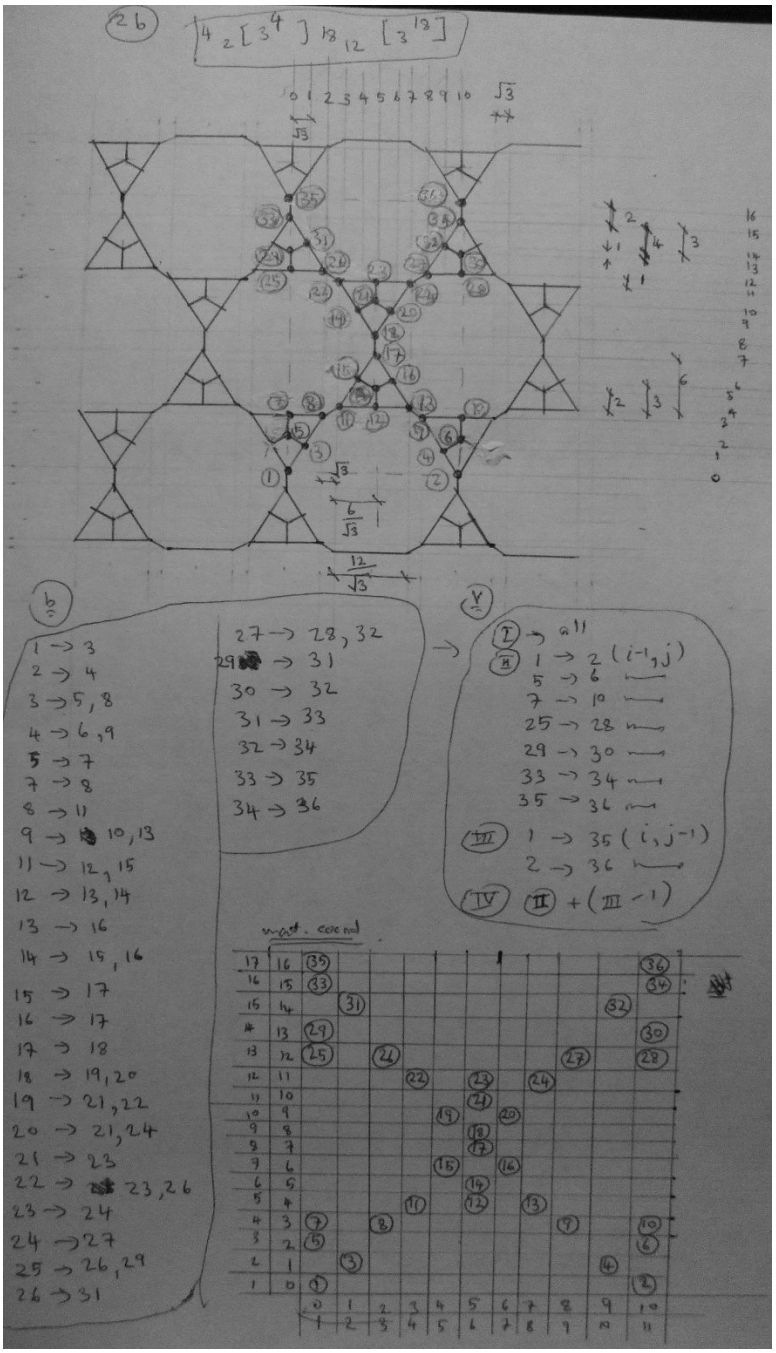

22
$5_2[3^5]12_{12}[3^{12}]$
0 1 2 3 4 5 6 7 8
1 → 3,6
2 → 4,6
3 → 5,11
4 → 7,12
5 → 8
6 → 9
8 → 13
9 → 11,12
10 → 14
11 → 13
12 → 14
13 → 15
14 → 16
15 → 17
16 → 18
17 → 19,21
18 → 20,23
19 → 22,27
20 → 22,28
21 → 24
22 → 25
→ 27
25 → 29,30
26 → 28
27 → 29
28 → 30
29 → 31
30 → 32
31 → 33
32 → 34
I all
II 5 → 7 (i-1, j)
8 → 10
21 → 23
24 → 26
III 1 → 33 (i, j-1)
2 → 34
IV V + III
mat. + coord.
int.
15 → 14 (i-1

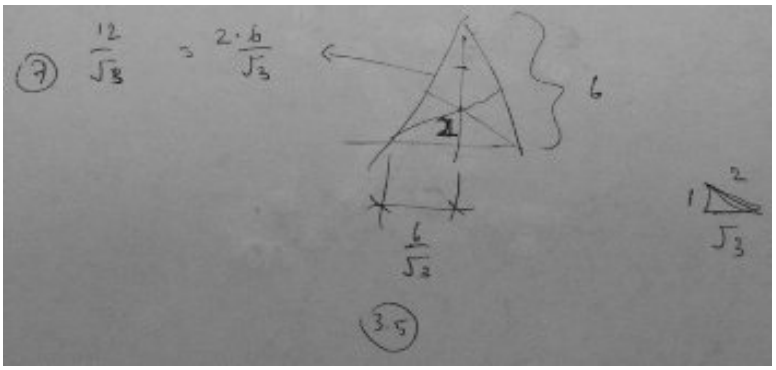

(23) $5_4[3^5]7_4[3^7]$ II
0 1 2 3 4 5 6 7 8 9 10
2
$\frac{1}{2}$
$\frac{\sqrt{3}}{2}$
(i) 1 → 4, 6
2 → 5, 7
3 → 8, 9
4 → 7, 14
5 → 8
6 → 10
7 → 11
8 → 12
10 → 14
11 → 16, 17
12 → 15, 17
13 → 18
14 → 16
15 → 18
16 → 19
17 → 20
18 → 21
(ii) (I) all
(II) 6 → 9 (i-1, j)
10 → 13
(III) 1 → 19 (i, j-1)
2 → 20
3 → 21
try (II) + (III)
mat. + cond.
int.
5 → 15 (i, j-1)

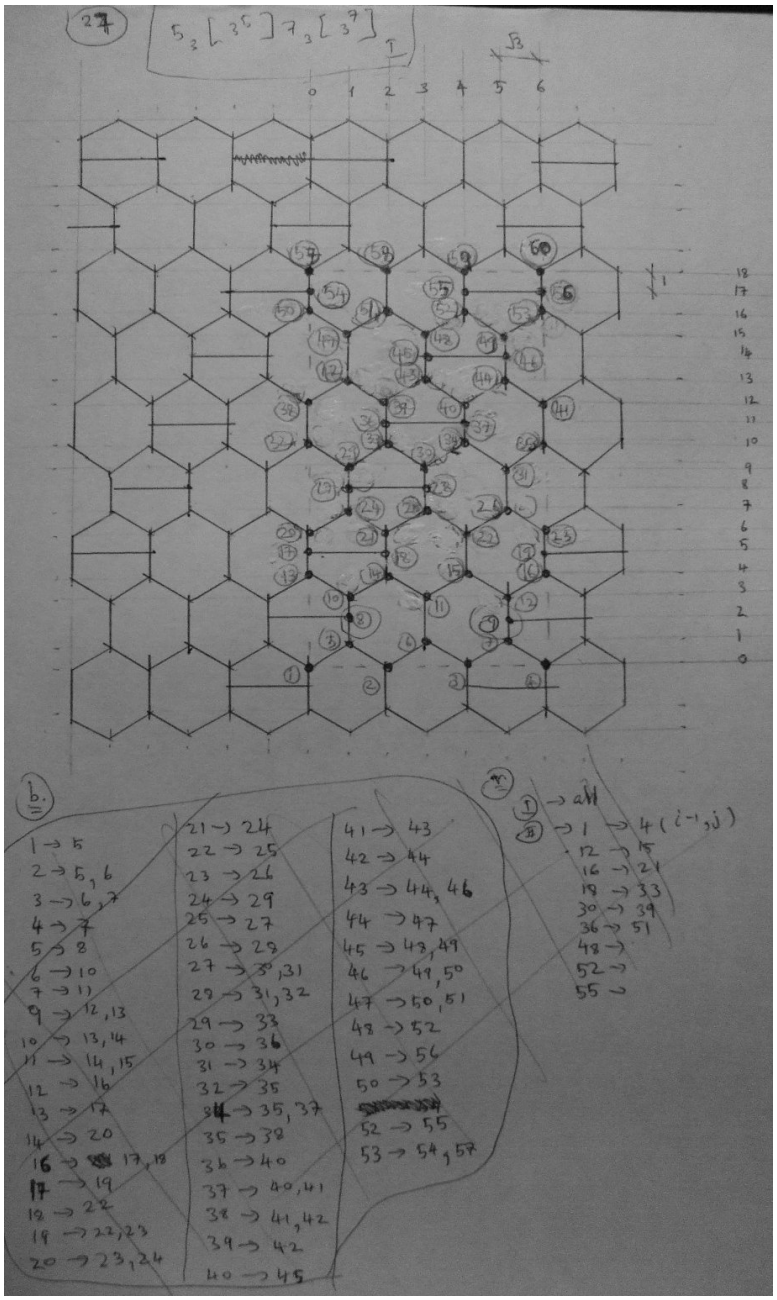

24
$5_3[3^5]8_6[3^8]$
I
0 1 2 3 4 5 6 7 8
1 → 3, 6
2 → 4, 7
3 → 5, 11
4 → 6
5 → 8
6 → 9
8 → 13
9 → 11, 14
10 → 12
11 → 13
12 → 14
13 → 15
14 → 16
15 → 17, 20
16 → 18, 21
17 → 19
18 → 20, 26
19 → 22
20 → 23
21 → 24
22 → [illegible], 27
23 → 25, 28
24 → 26
25 → 27
26 → 28
27 → 29
28 → 30
5 → 7 (i-1, j)
8 → 10
19 → 21
21 → 24
1 → 29 (i, j-1)
2 → 30
4 → 25 (i, j-1)
17 → 12 (i-1, j)
Always valid?

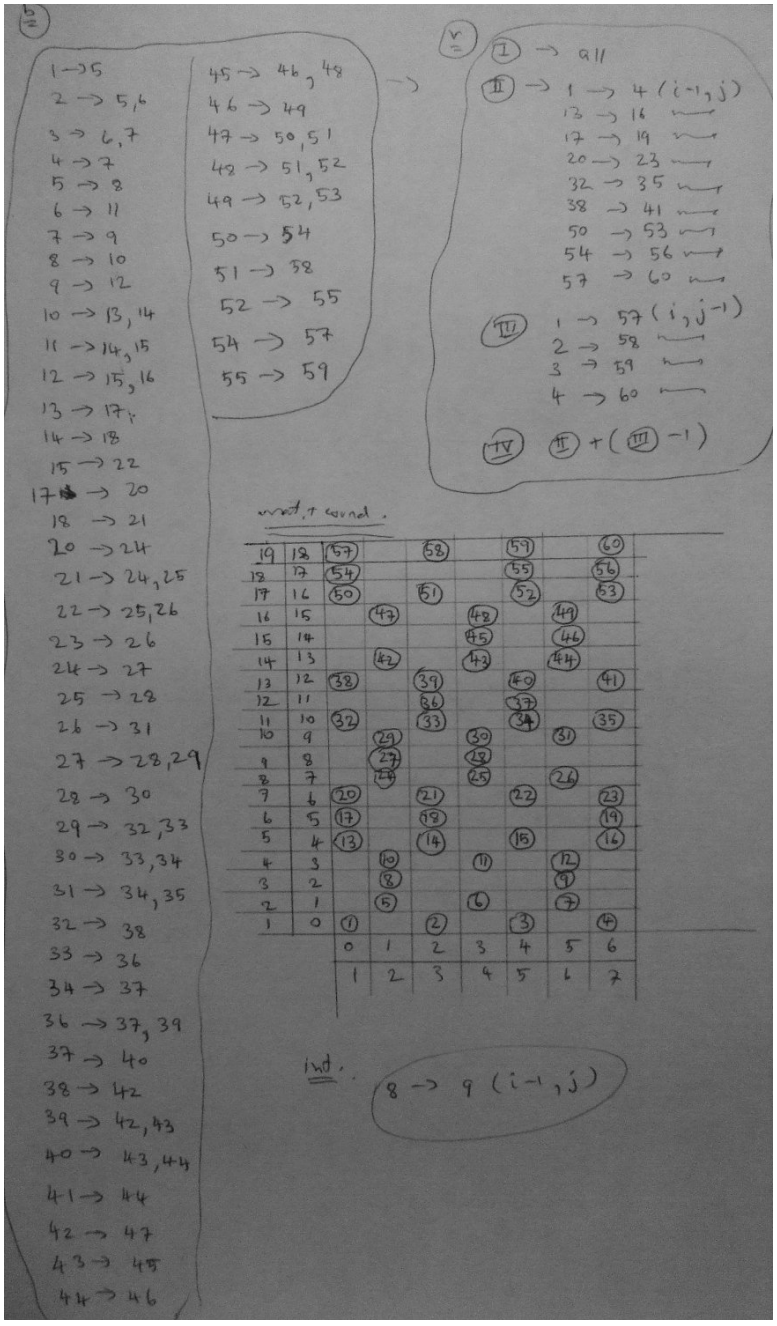

25
$4_2\,[3^4]\,12_6\,[3^{12}]$
1 → 2, 6
2 → 3, 5
3 → 7
5 → 6, 7
6 → 8
7 → 8
8 → 9
9 → 10, 11
11 → 12, 14
12 → 13
13 → 14, 15
14 → 16
15 → 16
16 → 17
17 → 20, 21
I → a 11
II 1 → 4 (i⁻¹, j)
18 → 21
III 1 → 18 (i, j⁻¹)
2 → 19
3 → 20
4 → 21
IV II − (III − 1)
10 → 12 (i⁻¹, j)
10 → 15

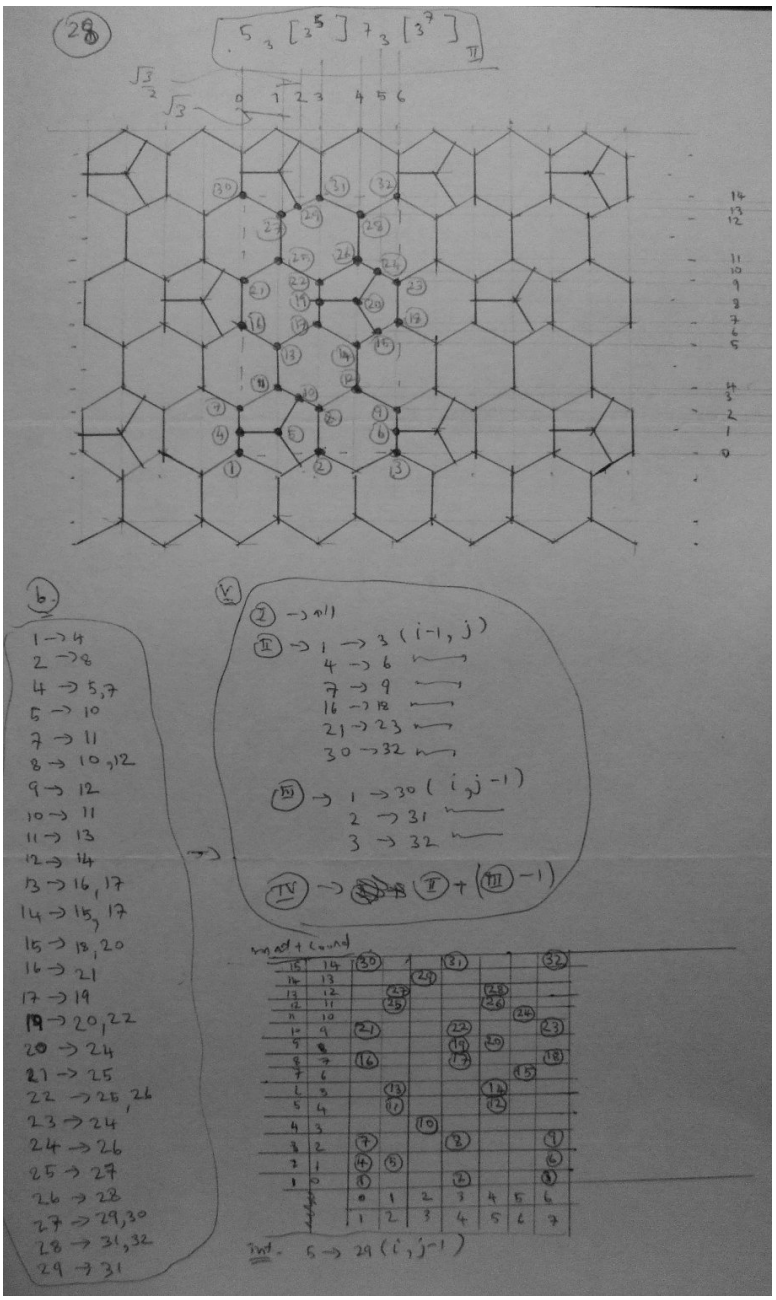


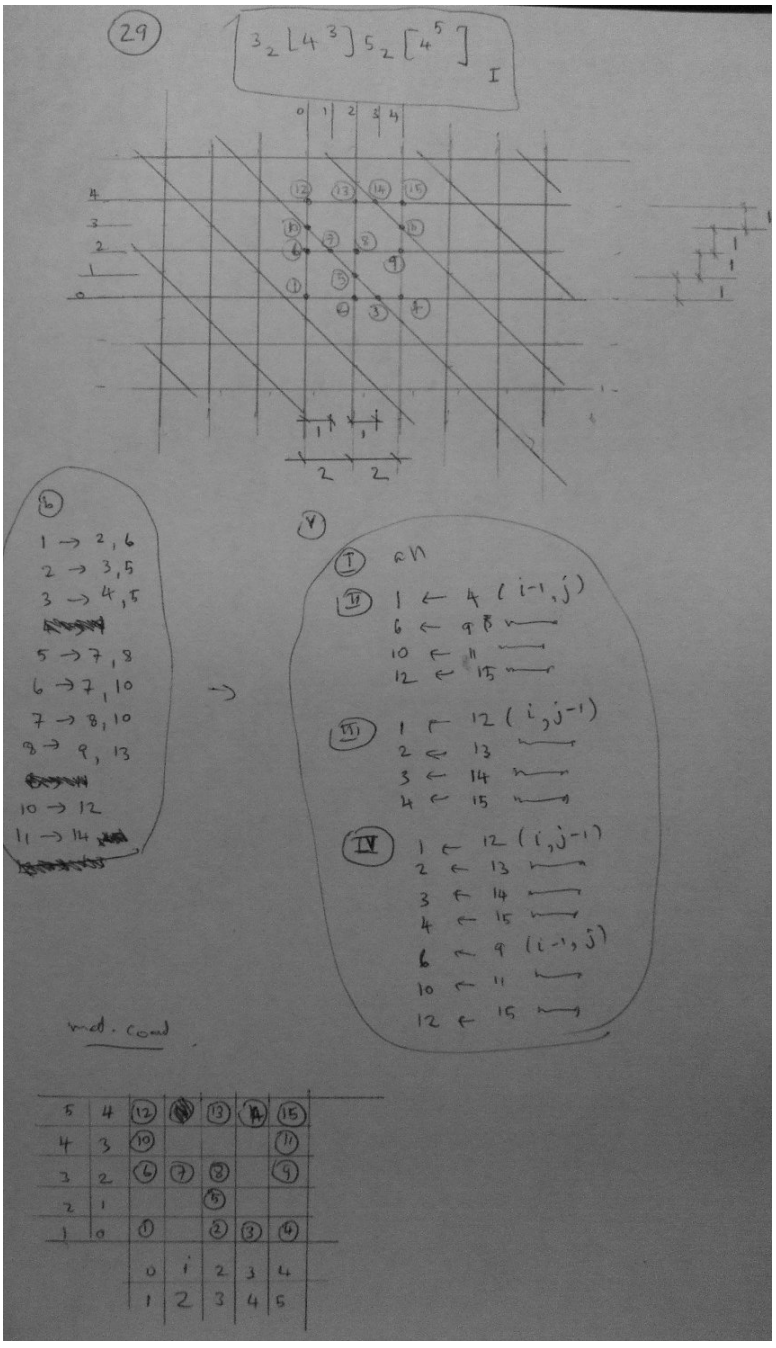

26
$4_2[3^4]\,18\,12\,[3^{12}]$
1 → 3
2 → 4
3 → 5, 8
4 → 6, 9
5 → 7
7 → 8
8 → 11
9 → 10, 13
11 → 12, 15
12 → 13, 14
13 → 16
14 → 15, 16
15 → 17
16 → 17
17 → 18
18 → 19, 20
19 → 21, 22
20 → 21, 24
21 → 23
22 → 23, 26
23 → 24
24 → 27
25 → 26, 29
26 → 31
27 → 28, 32
29 → 31
30 → 32
31 → 33
32 → 34
33 → 35
34 → 36
I → all
II 1 → 2 (i-1, j)
5 → 6
7 → 10
25 → 28
29 → 30
33 → 34
35 → 36
III 1 → 35 (i, j-1)
2 → 36
IV II + (III - 1)

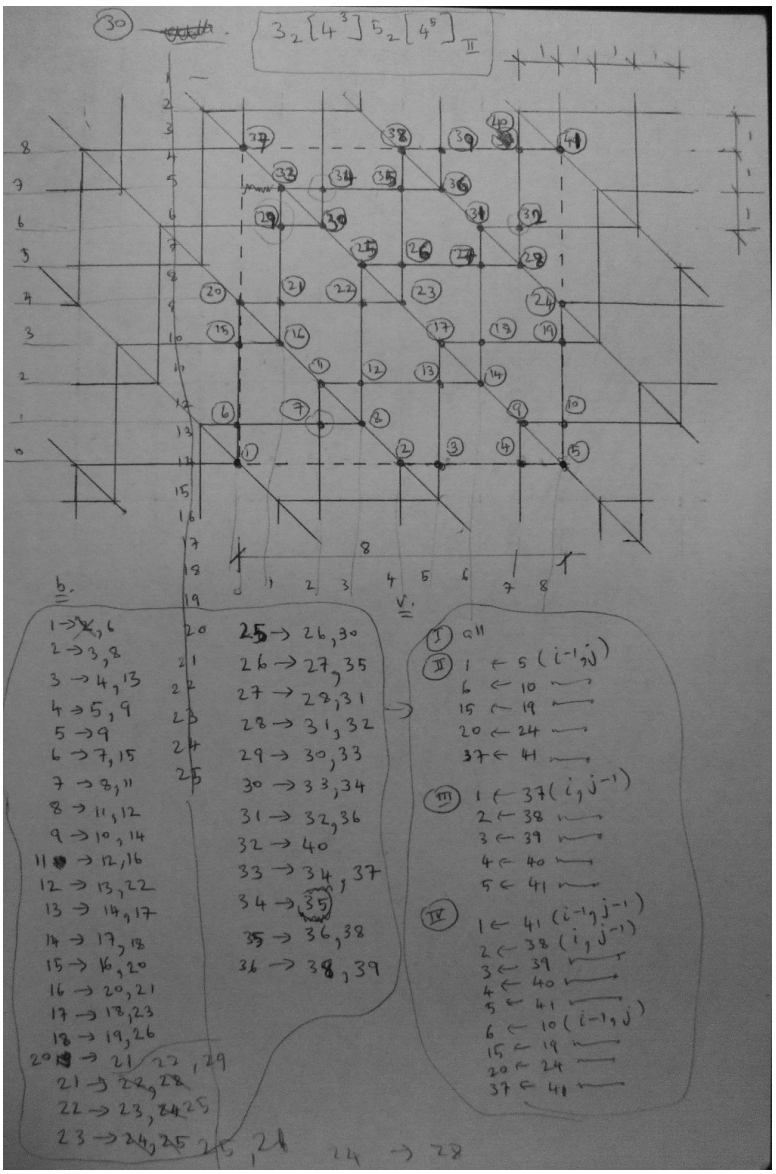

7
12/√3 = 2·b/√3
6
2
b/√2
1
2
√3
3.5

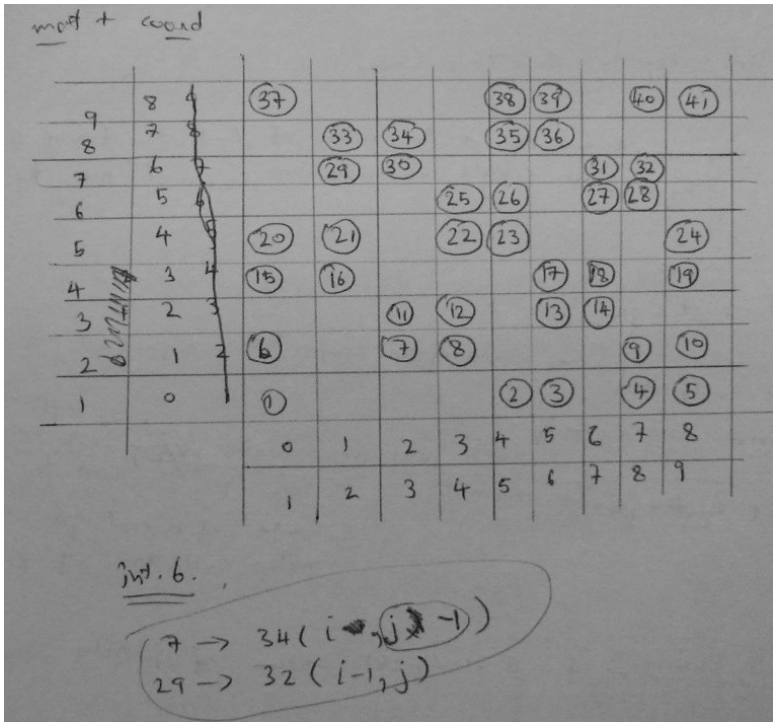

b.
1 → 5
2 → 5, 6
3 → 6, 7
4 → 7
5 → 8
6 → 10
7 → 11
9 → 12, 13
10 → 13, 14
11 → 14, 15
12 → 16
13 → 17
14 → 20
16 → 17, 18
17 → 19
18 → 22
19 → 22, 23
20 → 23, 24
21 → 24
22 → 25
23 → 26
24 → 29
25 → 27
26 → 28
27 → 30, 31
29 → 31, 32
29 → 33
30 → 36
31 → 34
32 → 35
34 → 35, 37
35 → 38
36 → 40
37 → 40, 41
38 → 41, 42
39 → 42
40 → 45
41 → 43
42 → 44
43 → 44, 46
44 → 47
45 → 43, 49
46 → 49, 50
47 → 50, 51
48 → 52
49 → 56
50 → 53
52 → 55
53 → 54, 57

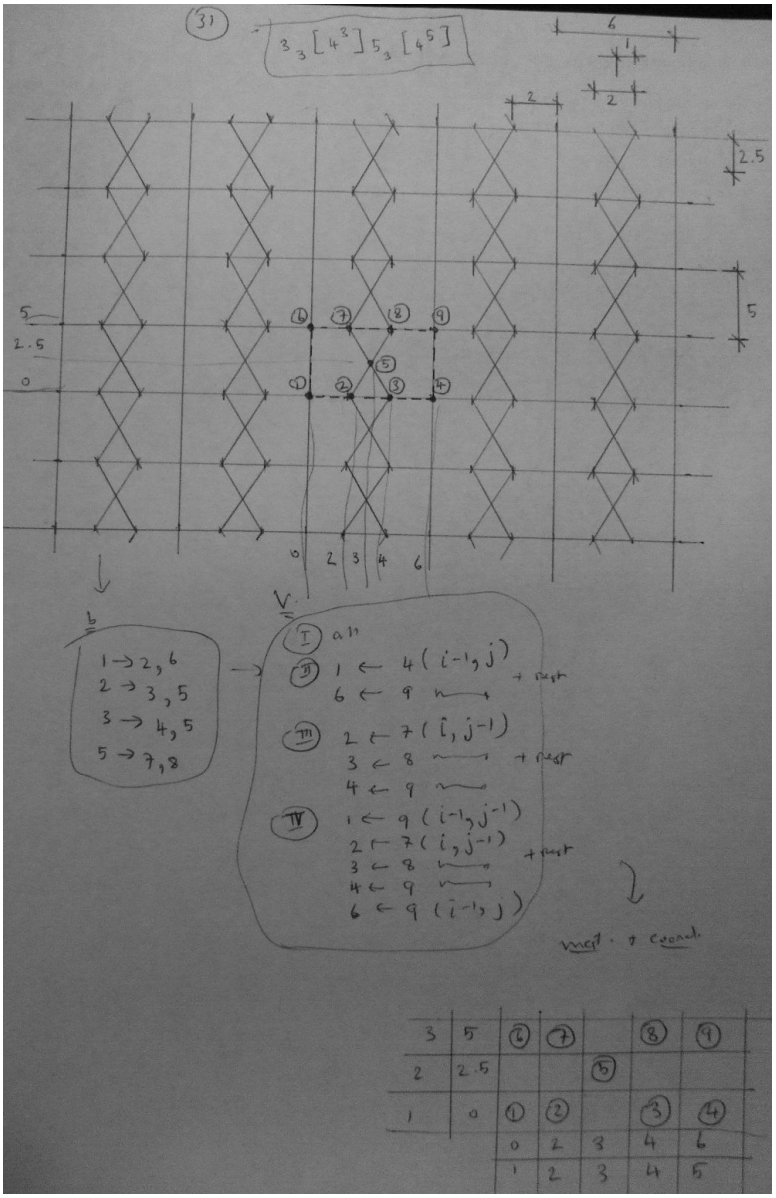

1 → 5
2 → 5,6
3 → 6,7
4 → 7
5 → 8
6 → 11
7 → 9
8 → 10
9 → 12
10 → 13,14
11 → 14,15
12 → 15,16
13 → 17
14 → 18
15 → 22
17 → 20
18 → 21
20 → 24
21 → 24,25
22 → 25,26
23 → 26
24 → 27
25 → 28
26 → 31
27 → 28,29
28 → 30
29 → 32,33
30 → 33,34
31 → 34,35
32 → 38
33 → 36
34 → 37
36 → 37,39
37 → 40
38 → 42
39 → 42,43
40 → 43,44
41 → 44
42 → 47
43 → 45
44 → 46
45 → 46,48
46 → 49
47 → 50,51
48 → 51,52
49 → 52,53
50 → 54
51 → 38
52 → 55
54 → 57
55 → 59
I → all
II → 1 → 4 (i-1, j)
13 → 16
17 → 19
20 → 23
32 → 35
38 → 41
50 → 53
54 → 56
57 → 60
III 1 → 57 (i, j-1)
2 → 58
3 → 59
4 → 60
IV II + (III - 1)
mod. + round.
int.
8 → 9 (i-1, j)

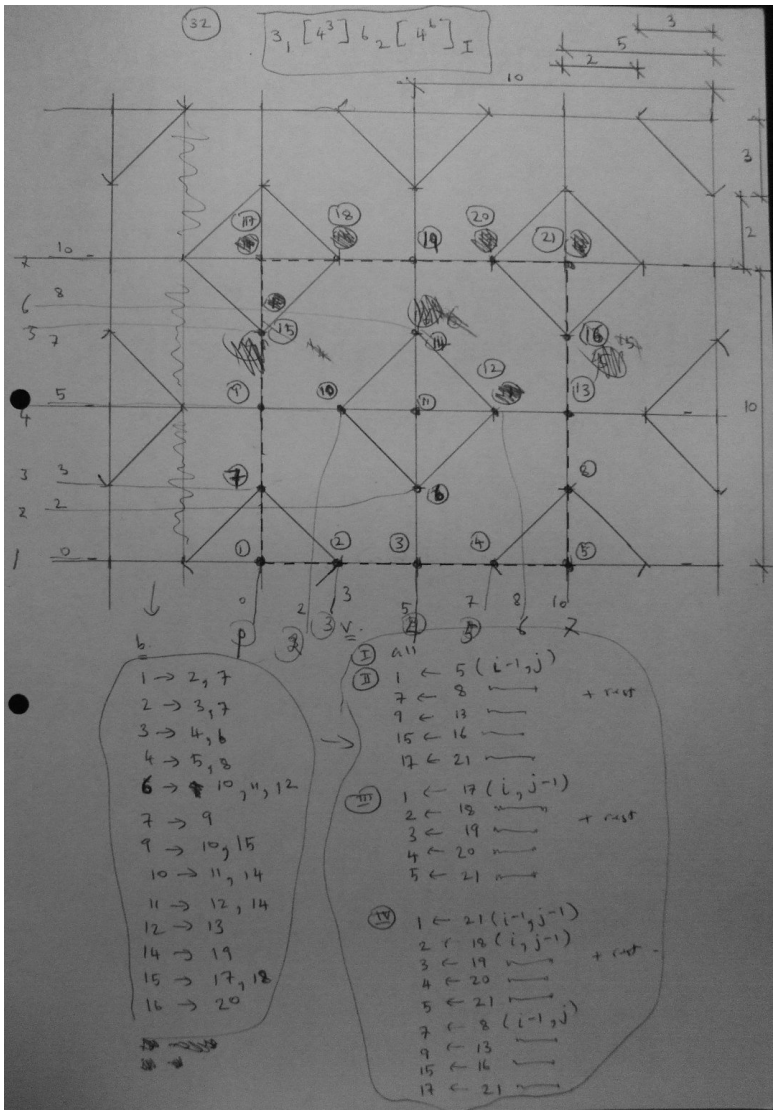


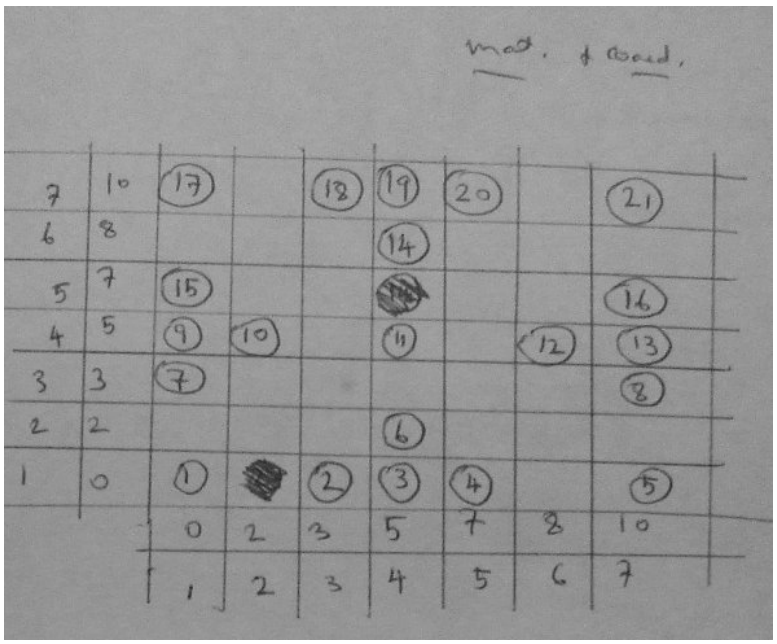

29
$3_2[4^3]5_2[4^5]$
I
0 1 2 3 4
4
3
2
1
0
2 2
6
1 → 2, 6
2 → 3, 5
3 → 4, 5
5 → 7, 8
6 → 7, 10
7 → 8, 10
8 → 9, 13
10 → 12
11 → 14
7
I
II
1 ← 4 (i-1, j)
6 ← 9
10 ← 11
12 ← 15
III
1 ← 12 (i, j-1)
2 ← 13
3 ← 14
4 ← 15
IV
1 ← 12 (i, j-1)
2 ← 13
3 ← 14
4 ← 15
6 ← 9 (i-1, j)
10 ← 11
12 ← 15
mod. cond
5 4 12 13 15
4 3 10 11
3 2 6 7 8 9
2 1 5
1 0 1 2 3 4
0 1 2 3 4
1 2 3 4 5

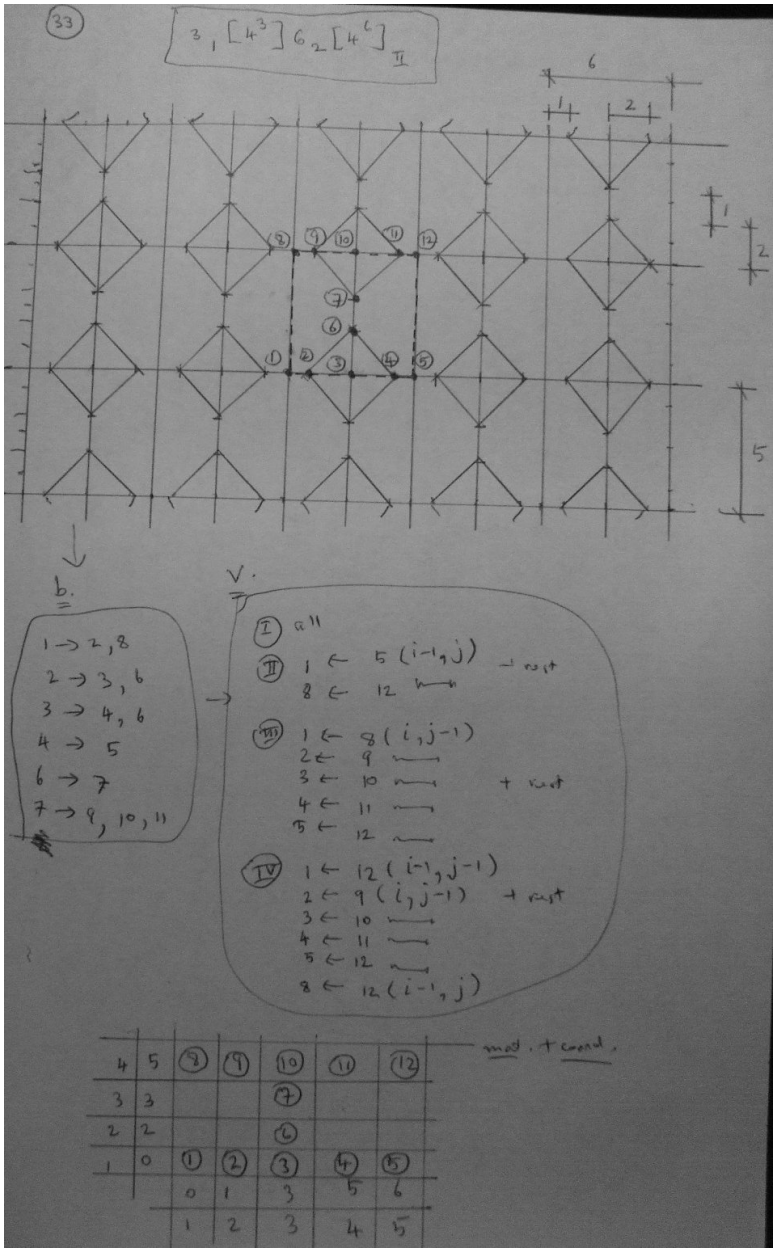

30
$3_2[4^3]5_2[4^5]$
b.
v.
1 → 5, 6
2 → 3, 8
3 → 4, 13
4 → 5, 9
5 → 9
6 → 7, 15
7 → 8, 11
8 → 11, 12
9 → 10, 14
11 → 12, 16
12 → 13, 22
13 → 14, 17
14 → 17, 18
15 → 16, 20
16 → 20, 21
17 → 18, 23
18 → 19, 26
20 → 21, 22, 29
21 → 22, 28
22 → 23, 24, 25
23 → 24, 25
25 → 26, 30
26 → 27, 35
27 → 29, 31
28 → 31, 32
29 → 30, 33
30 → 33, 34
31 → 32, 36
32 → 40
33 → 34, 37
34 → 35
35 → 36, 38
36 → 38, 39
I all
II 1 ← 5 (i-1, j)
6 ← 10
15 ← 19
20 ← 24
37 ← 41
III 1 ← 37 (i, j-1)
2 ← 38
3 ← 39
4 ← 40
5 ← 41
IV 1 ← 41 (i-1, j-1)
2 ← 38 (i, j-1)
3 ← 39
4 ← 40
5 ← 41
6 ← 10 (i-1, j)
15 ← 19
20 ← 24
37 ← 41

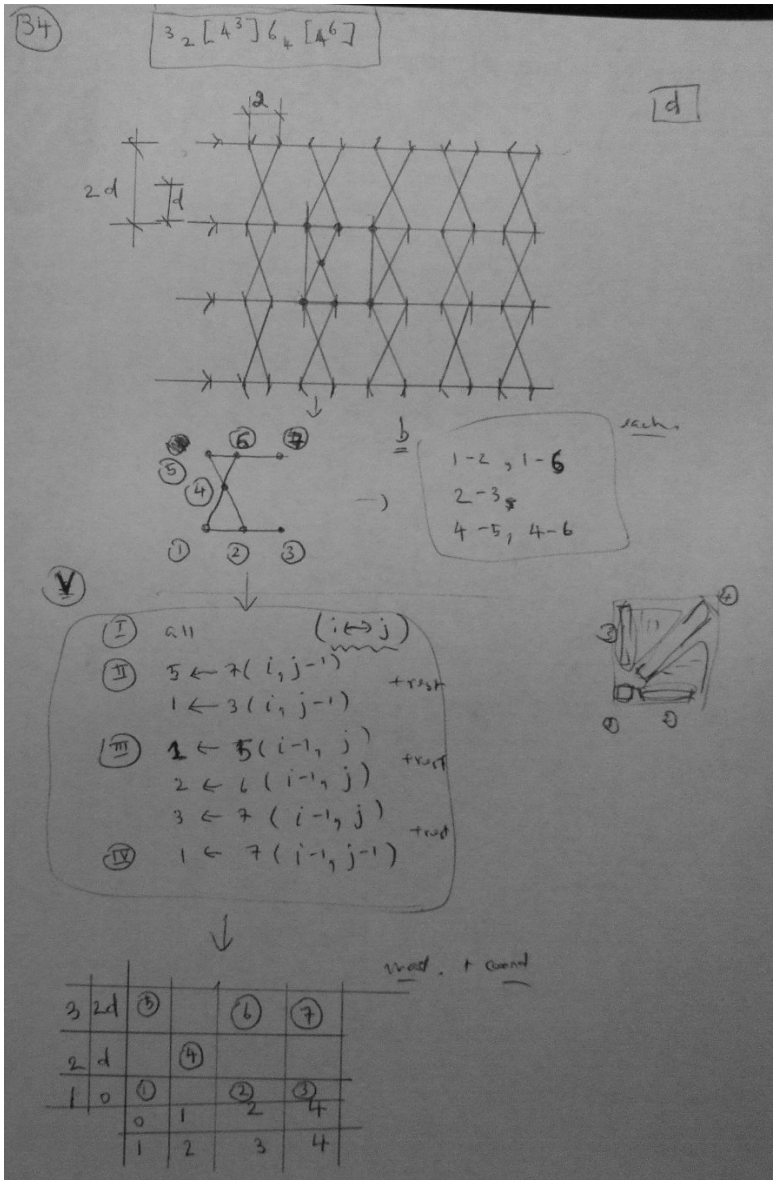

mat + coord
37
38
39
40
41
33
34
35
36
29
30
31
32
25
26
27
28
20
21
22
23
24
15
16
17
18
19
11
12
13
14
6
7
8
9
10
1
2
3
4
5
0 1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8 9
int. 6.
7 -> 34 (i , j - 1)
29 -> 32 (i-1 , j)

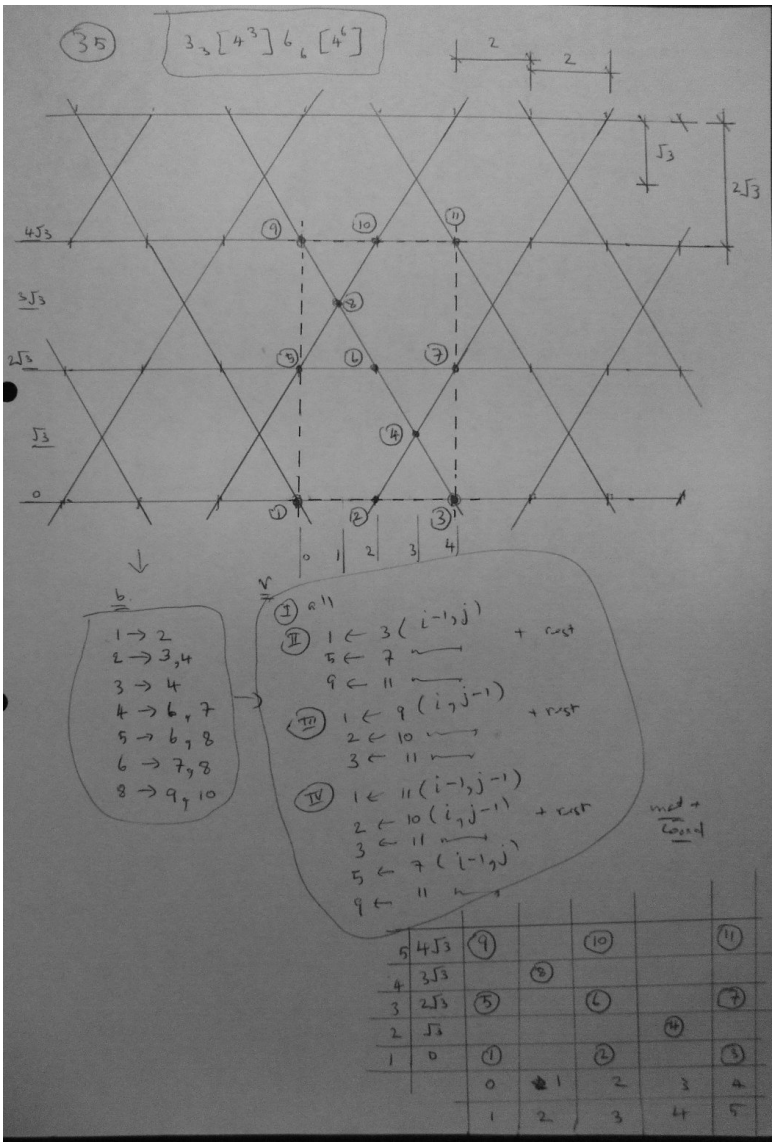

31
3 3 [4 3] 5 3 [4 5]
6
1
2
2
2.5
5
2.5
0
0 2 3 4 6
V
I an
II 1 ← 4 (i-1, j)
6 ← 9
III 2 ← 7 (i, j-1)
3 ← 8
4 ← 9
IV 1 ← 9 (i-1, j-1)
2 ← 7 (i, j-1)
3 ← 8
4 ← 9
6 ← 9 (i-1, j)
1 → 2, 6
2 → 3, 5
3 → 4, 5
5 → 7, 8
3 5
2 2.5
1 0
0 2 3 4 6
1 2 3 4 5

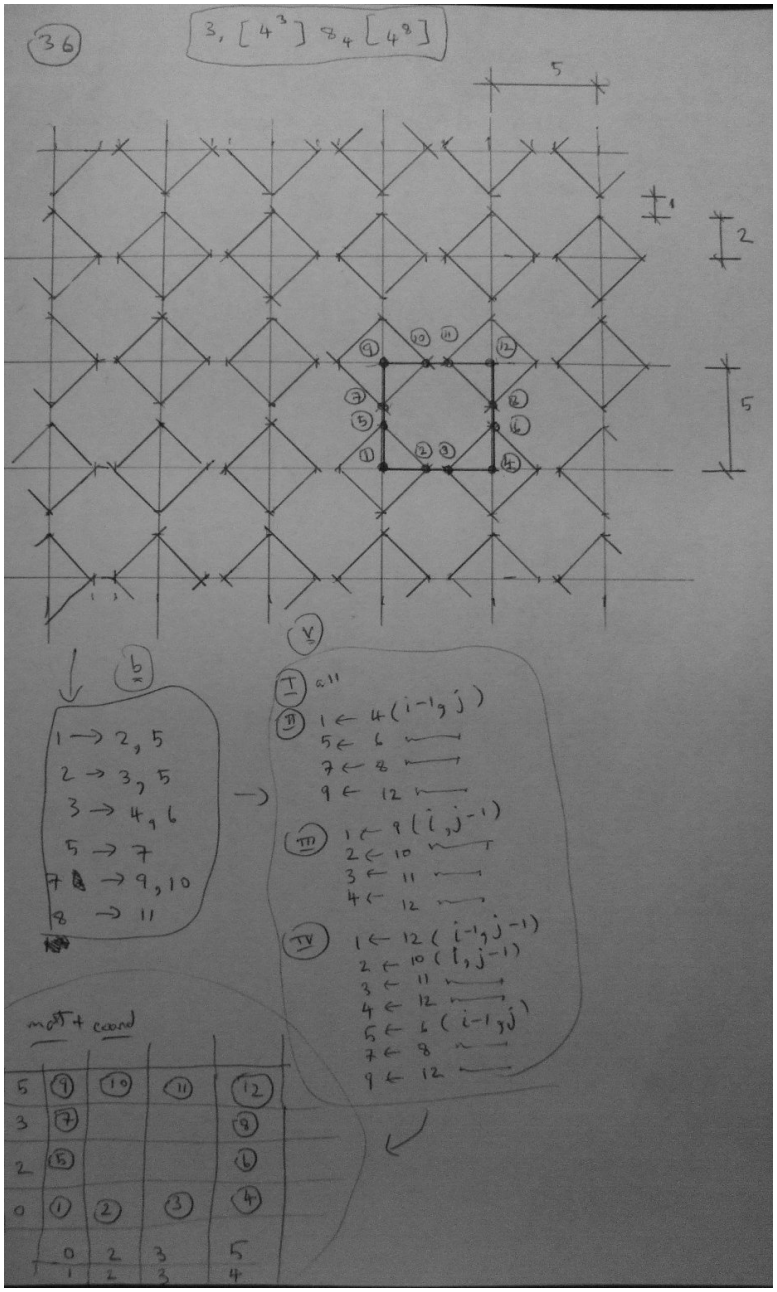

32
3, [4^3] 6 2 [4^6] I
1 → 2, 7
2 → 3, 7
3 → 4, 6
4 → 5, 8
6 → 10, 11, 12
7 → 9
9 → 10, 15
10 → 11, 14
11 → 12, 14
12 → 13
14 → 19
15 → 17, 18
16 → 20
I all
II 1 ← 5 (i-1, j)
7 ← 8
9 ← 13
15 ← 16
17 ← 21
III 1 ← 17 (i, j-1)
2 ← 18
3 ← 19
4 ← 20
5 ← 21
IV 1 ← 21 (i-1, j-1)
2 ← 18 (i, j-1)
3 ← 19
4 ← 20
5 ← 21
7 ← 8 (i-1, j)
9 ← 13
15 ← 16
17 ← 21

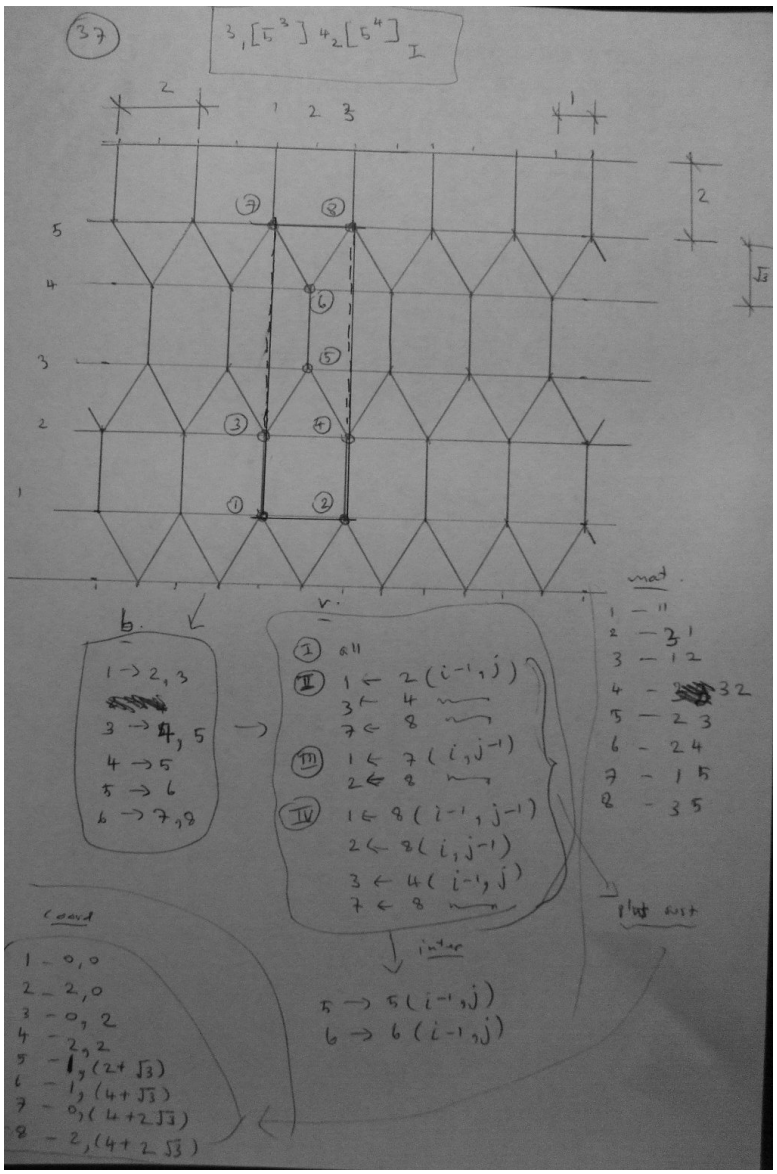

mat. & board.
7 10 17 18 19 20 21
6 8 14
5 7 15 16
4 5 9 10 11 12 13
3 3 7 8
2 2 6
1 0 1 2 3 4 5
0 2 3 5 7 8 10
1 2 3 4 5 6 7

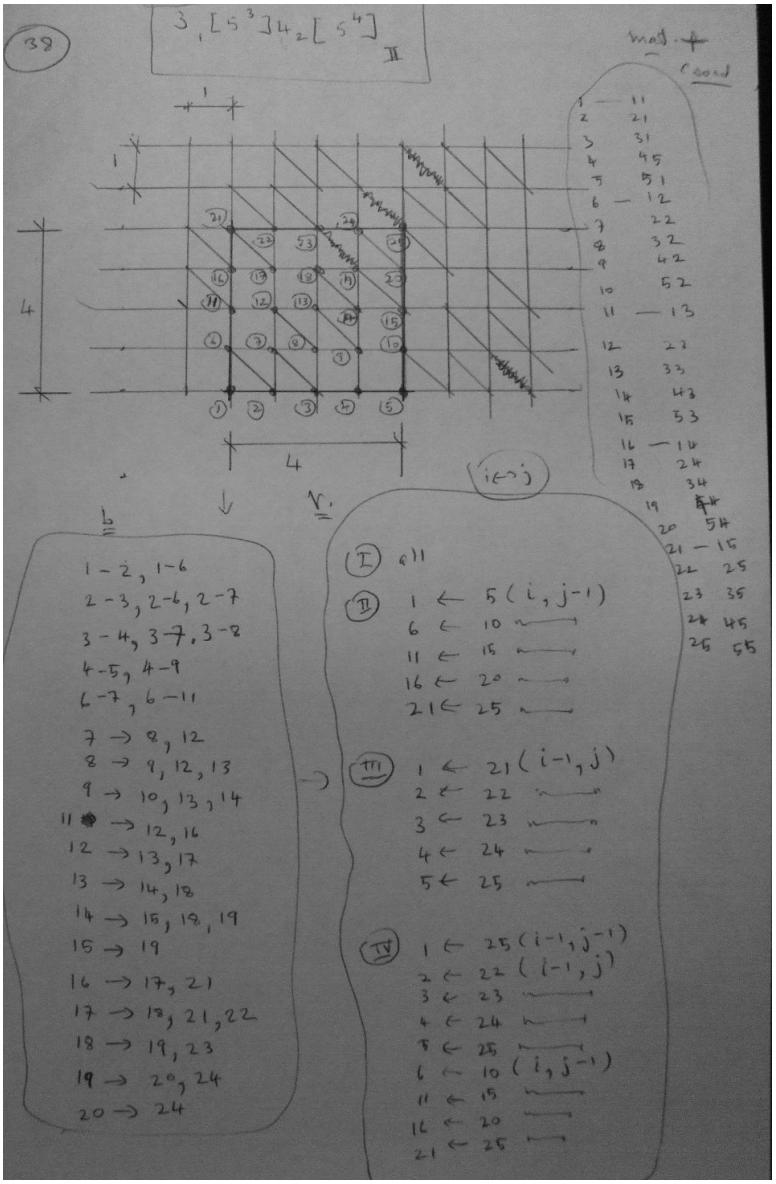

33
3, [4^3] 6, [4^6] II
6
1
2
1
2
5
b.
1 → 2, 8
2 → 3, 6
3 → 4, 6
4 → 5
6 → 7
7 → 9, 10, 11
V.
I all
II 1 ← 5 (i-1, j) + rest
8 ← 12
III 1 ← 8 (i, j-1)
2 ← 9
3 ← 10 + rest
4 ← 11
5 ← 12
IV 1 ← 12 (i-1, j-1)
2 ← 9 (i, j-1) + rest
3 ← 10
4 ← 11
5 ← 12
8 ← 12 (i-1, j)
mod. + round.

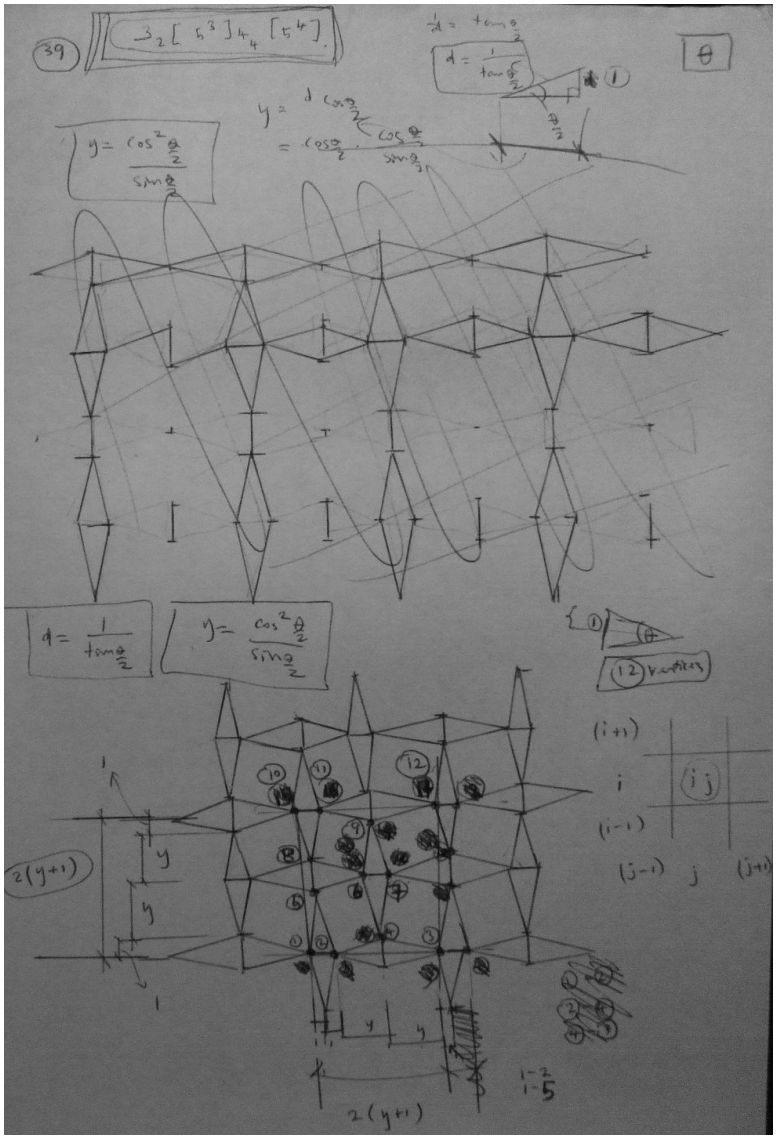

B4
$3_2[4^3]6_4[4^6]$
d
2
2d
d
b
1-2, 1-6
2-3
4-5, 4-6
(i↔j)
all
$5 \leftarrow 7(i, j^{-1})$
$1 \leftarrow 3(i, j^{-1})$
$1 \leftarrow 5(i^{-1}, j)$
$2 \leftarrow 6(i^{-1}, j)$
$3 \leftarrow 7(i^{-1}, j)$
$1 \leftarrow 7(i^{-1}, j^{-1})$


35
$3_3[4^3]\,6_6[4^6]$
2
2
$\sqrt{3}$
$2\sqrt{3}$
$4\sqrt{3}$
$3\sqrt{3}$
$2\sqrt{3}$
$\sqrt{3}$
0
0 1 2 3 4
b
1 → 2
2 → 3,4
3 → 4
4 → 6, 7
5 → 6, 8
6 → 7, 8
8 → 9, 10
v
I a 11
II 1 ← 3 (i-1, j)
5 ← 7
9 ← 11
+ rest
III 1 ← 9 (i, j-1)
2 ← 10
3 ← 11
+ rest
IV 1 ← 11 (i-1, j-1)
2 ← 10 (i, j-1)
3 ← 11
5 ← 7 (i-1, j)
9 ← 11
+ rest
mod + coord


36
3, [4^3] 8_4 [4^8]
5
2
5
1 → 2, 5
2 → 3, 5
3 → 4, 6
5 → 7
7 → 9, 10
8 → 11
1 ← 4 (i-1, j)
5 ← 6
7 ← 8
9 ← 12
1 ← 9 (i, j-1)
2 ← 10
3 ← 11
4 ← 12
1 ← 12 (i-1, j-1)
2 ← 10 (i, j-1)
3 ← 11
4 ← 12
5 ← 6 (i-1, j)
7 ← 8
9 ← 12
matt + coord


37
3,[5^3] 4,[5^4] I
mat.
coord
init
1 → 2, 3
3 → 4, 5
4 → 5
5 → 6
6 → 7, 8
5 → 5(i⁻¹, j)
6 → 6(i⁻¹, j)


38
3, [5^3] 4, [5^4]
II
mod
coord
1 — 11
2 21
3 31
4 45
5 51
6 — 12
7 22
8 32
9 42
10 52
11 — 13
12 23
13 33
14 43
15 53
16 — 14
17 24
18 34
19 54
20 54
21 — 15
22 25
23 35
24 45
25 55
i ↔ j
I all
II 1 ← 5 (i, j-1)
6 ← 10
11 ← 15
16 ← 20
21 ← 25
III 1 ← 21 (i-1, j)
2 ← 22
3 ← 23
4 ← 24
5 ← 25
IV 1 ← 25 (i-1, j-1)
2 ← 22 (i-1, j)
3 ← 23
4 ← 24
5 ← 25
6 ← 10 (i, j-1)
11 ← 15
16 ← 20
21 ← 25
1 - 2, 1-6
2 - 3, 2-6, 2-7
3 - 4, 3-7, 3-8
4 - 5, 4-9
6 - 7, 6-11
7 → 8, 12
8 → 9, 12, 13
9 → 10, 13, 14
11 → 12, 16
12 → 13, 17
13 → 14, 18
14 → 15, 18, 19
15 → 19
16 → 17, 21
17 → 18, 21, 22
18 → 19, 23
19 → 20, 24
20 → 24


39
$3_2[5^3]4_4[5^4]$.
$\frac{1}{d} = \tan\frac{\theta}{2}$
$d = \frac{1}{\tan\frac{\theta}{2}}$
$y = d\cos\frac{\theta}{2}$
$= \cos\frac{\theta}{2} \cdot \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$
$y = \frac{\cos^2\frac{\theta}{2}}{\sin\frac{\theta}{2}}$
$d = \frac{1}{\tan\frac{\theta}{2}}$
$y = \frac{\cos^2\frac{\theta}{2}}{\sin\frac{\theta}{2}}$
12 vertices
(i+1)
i
(i,j)
(i-1)
(j-1)
j
(j+1)
2(y+1)
y
y
2(y+1)